

## REDUCTION IN NON-LINEARITY IN POSITION-SENSITIVE MWPCs\*

E. Mathieson, Leicester University, Leicester LE1 7RH, England

G. C. Smith, Brookhaven National Laboratory, Upton, New York 11973, U.S.A.

Abstract

Measurements of differential non-linearity have been made, with a small multiwire proportional chamber, for two special cathode strip geometries. The results have been compared with that for conventional strip geometry, and with theoretical predictions. With the ratio of sampling node separation to anode, cathode separation at the exceptionally large value 2.4, a peak-to-peak differential non-linearity of 72% was observed for the conventional geometry. The first special geometry employed two intermediate strips between the readout strips. By using near optimum capacitive coupling between strips, a differential non-linearity of about 4% has been achieved at the same ratio 2.4. In the second special geometry, cathode strips with optimum depth zigzag shaped boundaries yielded an experimental differential non-linearity of about 7%.

1. Introduction

As a position-sensitive detector the MWPC has found major applications over the whole range of radiation types, from high-energy particles to low-energy x-rays. For position measurement parallel to the anode wire direction, using cathode induced charge signals, remarkable performance figures with respect to resolution and linearity have now been achieved. This present paper is concerned with the reduction in non-linearity, parallel to the anode wire direction, that can be obtained under what would normally be regarded as seriously unfavourable sampling conditions.

The cathode induced charge has a bell-shaped distribution with a full width at half-maximum of roughly 1.5 times the anode-cathode spacing. Encoding non-linearity will occur when the width of the cathode charge sampling strips is too large to form, at all positions, an accurate value of the centroid of this distribution. A series of early studies (see for example Refs. [1], [2], [3]) has shown that this sampling differential non-linearity is negligible,  $\sim \pm 1\%$ , when the strip width is equal to or less than about 0.8 times the anode, cathode spacing. There may however be constraints which prevent this small value being reached. Specifically, in constructing a chamber for high resolution two-dimensional imaging, a small anode wire spacing must be employed and then, in order that a large enough cathode signal be produced, the anode, cathode separation,  $h$ , must be roughly of the same value. Thus the sampling width,  $w_r$ , must be correspondingly small. However, in certain situations, this may not be convenient, or even possible. For example, if the position encoding is to employ a lumped-parameter delay line, which is envisaged for a particular application we have in small angle x-ray scattering, then the number of delay line taps has been predetermined; an arbitrary increase in number is not possible.

Several methods have been proposed to overcome sampling non-linearity due to too large a value of the ratio  $w_r/h$ . For example, for the basic multi-channel, 'centre-of-gravity' method, resistive interpolation may be employed [4], and for the multi-channel, centroid-finding filter method [2], a special inter-

\*This research was supported in part by the U. S. Department of Energy: Contract No. DE-AC02-76CH00016.

polating filter has been successfully used [3]. With delay-line position encoding these approaches are not possible; the two methods described below are considered mainly for use with a lumped parameter delay line. Application in other situations may however be also favourable.

2. Experimental Arrangement

The apparatus employed was essentially the same as that already described in a previous paper [5]. A multiwire chamber, anode, cathode spacing  $h = 1.59$  mm and anode wire pitch  $s = 2.82$  mm, containing A/20%CO<sub>2</sub>, was uniformly irradiated with 5.4 keV x-rays. The position sensing cathode was fabricated from 1/16 inch printed circuit board. The various cathode strip geometries which have been investigated were produced by standard print and etch techniques. The sampling width,  $w_r$ , on each cathode was 3.81 mm. An exaggeratedly large ratio,  $w_r/h = 2.40$ , was deliberately chosen for these studies to demonstrate the degree of reduction in non-linearity that can be achieved by the methods described below. The cathode was connected to a 40 tap delay line with 25 ns delay per section [6]. However, in the presentation of experimental data, the response from only a small part of the delay line is shown for each cathode strip geometry, since it was easier, and more expeditious, to print several cathode geometries on one cathode board.

3. Intermediate Strip Cathodes(i) Conventional cathode

Figure 1(a) shows schematically the cathode strip structure and connections to the delay line of a conventional strip cathode. The experimental uniform irradiation response (UIR) of this cathode, at  $w_r/h = 2.40$ , is shown below in Fig. 1(b). The measured peak-to-peak differential non-linearity (DFNL) is 72%.

For this chamber geometry, the Gatti empirical distribution of cathode charge [1] may be defined by the single parameter  $K_2 = 0.667$ , see Ref. [7,8]. A simple 'geometrical' calculation of DFNL for this situation yields 72%, in agreement with experiment. A more elaborate calculation of DFNL can be made by modelling theoretically the complete position-encoding system, with delay line clipping at 0.1  $\mu$ s and cross-over timing. This method, which of course is much lengthier, predicts 73%.

(ii) Single intermediate strip (SIS) cathode

In a previous approach [5] to the problem of increasing the ratio  $w_r/h$  without degrading linearity, a single intermediate strip, of width  $w_r/2$ , was capacitively coupled to the two adjacent read-out strips. A DFNL of 5% was achieved at  $w_r/h = 1.60$ , where the conventional strip DFNL was 23%.

(iii) Two intermediate strip (TIS) cathode

In this method, two capacitively coupled intermediate strips are employed, Fig. 2(a). This method was also briefly discussed in Ref. [5] where, using three equal value capacitors, the predicted performance was seen to be inferior to the SIS method

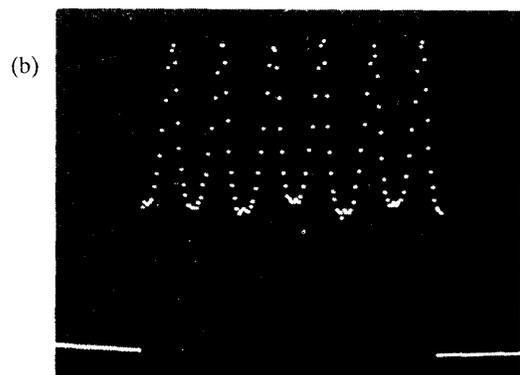
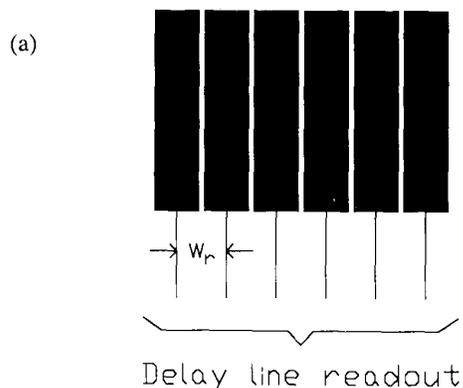


Fig. 1 (a) Schematic of conventional cathode strips and connections to delay line.  
 (b) Uniform irradiation response of MWPC with conventional cathode.  $w_r/h = 2.40$ , peak-to-peak differential non-linearity (DFNL) = 72%.

method. However it can be shown that if the central coupling capacitance,  $C_1'$  in Fig. 2(b), is correctly chosen then a very significant improvement in linearity can be achieved. This is essentially a limiting case of the capacitive tailoring scheme already described in Ref. [9]. The minimising of the DFNL with respect to the value of the ratio  $C_1'/C_1$  is shown in Fig. 3; this curve is for the values  $K_3 = 0.667$  and  $C_1/C_2 = 3.80$ . The manner in which the optimum value of  $C_1'/C_1$  depends on the ratio  $C_1/C_2$  is shown in Fig. 4.

Figure 5 demonstrates the remarkable decrease in non-linearity which can, in principle, be achieved by an optimised TIS cathode; the 72% for a conventional cathode may theoretically be reduced to less than 4%. Figure 5 also shows the predicted DFNL for the SIS arrangement under the same conditions. The predictions of DFNL in Fig. 5 were made using the simple 'geometrical' method of calculating the charge centroid. Using the full delay-line model, the predictions depend somewhat on the values chosen for delay line and amplifier rise-times. Results very close to those of Fig. 5 are obtained choosing the rise-time constants at 25 ns and 9 ns, respectively.

The predicted modulation in the UIR reverses in phase as the optimum value of  $C_1'/C_1$  is traversed. This is illustrated in Figs. 6(a) and (c). Figure 6(b) shows that, near the minimum DFNL position, the dominant modulation wavelength is  $w_r/2$ .

Accurate measurement of the very small capacitances  $C_1$  and  $C_2$  (in order to make realistic comparisons between predic-

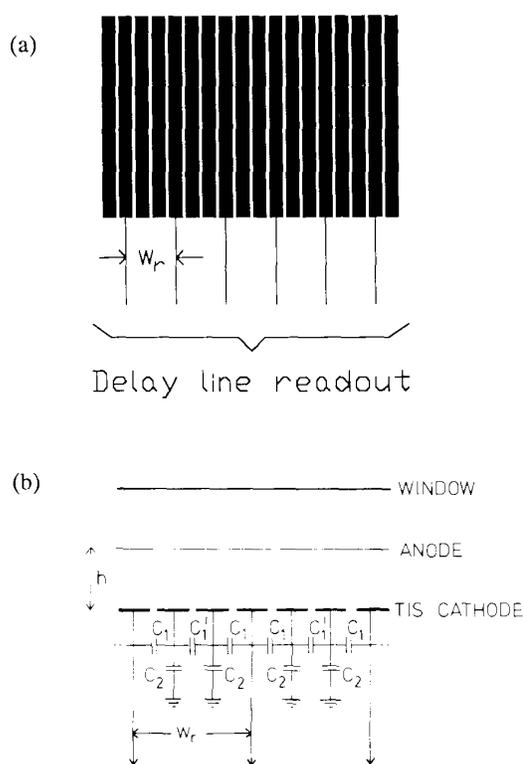


Fig. 2(a) Schematic of two-intermediate strip (TIS) cathode and connections to delay line.  
 (b) Capacitive coupling for two-intermediate strip cathode.  $C_1$  and  $C_2$  represent intrinsic capacitance.  $C_1'$  is achieved by addition of small chip capacitors.

tion and experiment) has proved to be rather difficult, as already discussed in Ref. [5].  $C_1$  is the intrinsic inter-strip capacitance which is incremented to  $C_1'$  by addition of a small chip capacitor. With no added capacitance, i.e.  $C_1'/C_1 = 1.0$ , the experimental DFNL is 20%. The value of  $C_1/C_2$  may therefore be estimated, from Fig. 5, as about 3.8, with a minimum achievable DFNL around 2.8%. An approximate value for  $C_1$  was obtained, as explained in Ref. [5], by connecting alternate strips together and then measuring, in situ, the capacitance between the two groups of strips. From this measurement a value  $C_1 = 1.3$  pF was estimated; the value of  $C_2$  would then be about 0.34 pF. A direct measurement of  $C_2$ , by replacing the TIS cathode by a continuous conducting rectangle of the same area and the same width, yielded 0.27 pF. The difference, 0.07 pF, can reasonably be accounted for by the presence between adjacent strips, in the operating system, of small 1 M $\Omega$  chip resistors. These resistors are necessary to stabilise the strip potentials. (A high resistance carbon coating was employed in the previous examination of the SIS arrangement.)

Experimental UIRs for two-intermediate strip cathodes are shown in Figs. 7(a) to (c). Figure 7(b), with an estimated  $C_1'/C_1 = 2.5$ , has DFNL  $\approx 4\%$ . The modulation is mainly at wavelength  $w_r/2$ , as predicted in Fig. 6(b). The phase change between Figs. 7(a) and (b), as in Fig. 6, is also demonstrated. Quoted figures of DFNL do not include the long wavelength modulation sometimes apparent in the uniform irradiation response. The origin of this is not understood at present, but it does not appear to be related to the special cathodes studied here.

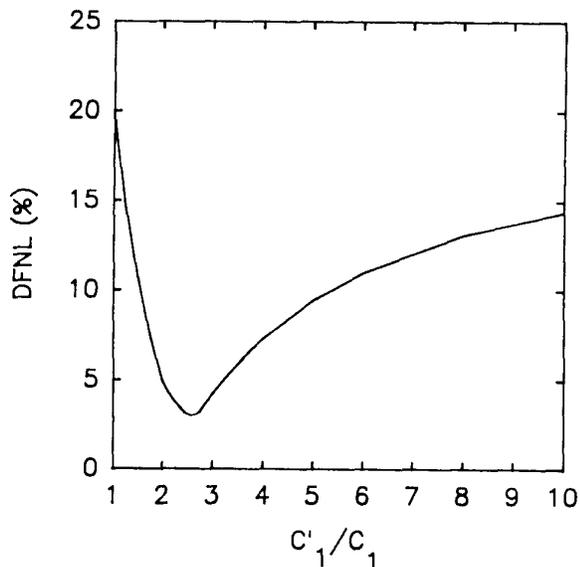


Fig. 3 Calculated differential non-linearity of TIS cathode as a function of  $C_1'/C_1$  (for  $C_1/C_2 = 3.80$ ,  $w_r/h = 2.40$  and  $K_3 = 0.667$ ). A minimum DFNL of 2.8% is predicted at  $C_1'/C_1 = 2.63$ .

A direct comparison between predicted and measured DFNLs is given in Table 1. The degree of agreement is considered quite satisfactory in view of the difficulties, described above, in the measurement of the capacitances.

In this table,  $f_q$  represents the fraction of induced cathode charge that is lost to ground; it is interesting to note that  $f_q$  depends only upon the ratio  $C_1/C_2$  and is independent of  $C_1'$ .

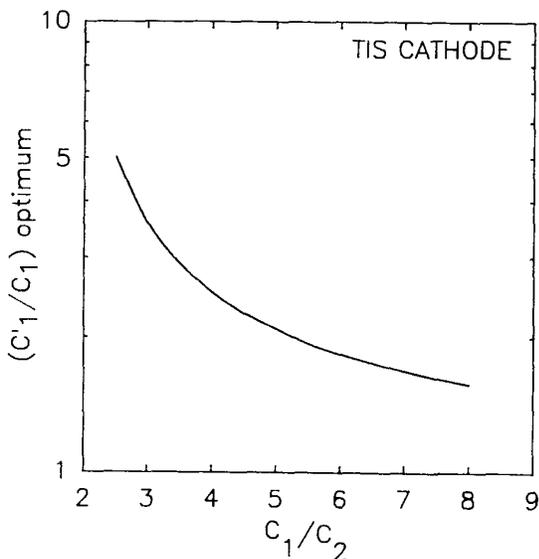


Fig. 4 Optimum value of  $C_1'/C_1$  as a function of  $C_1/C_2$ , for  $w_r/h = 2.40$  and  $K_3 = 0.667$ , (from simple 'geometrical' calculations).

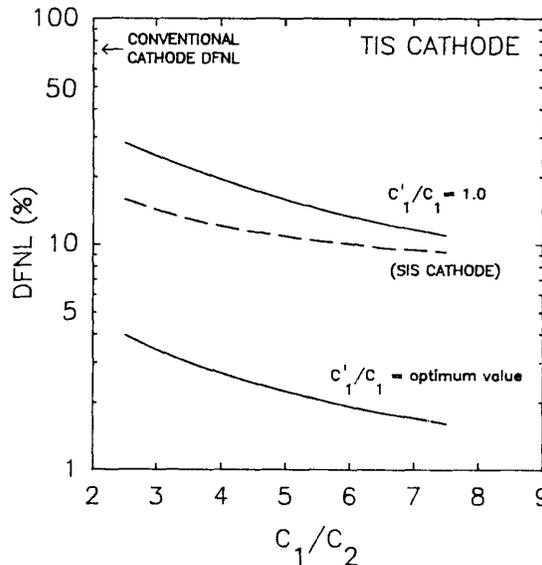


Fig. 5 Calculated differential non-linearity of TIS cathode as a function  $C_1/C_2$  (for  $w_r/h = 2.40$  and  $K_3 = 0.667$ ). The conventional cathode DFNL is 72%.

4. Zigzag strip (ZZS) cathodes.

Cathode strips with zigzag shaped boundaries have been successfully employed by Miki et al. [10] to reduce the number of readout nodes in a time projection chamber without losing positional resolution. Theoretical calculations have now been made of the differential non-linearity of such a cathode as a function of the depth of the zigzag waveform,  $f_x w_r$ , see Fig. 8. The period of the waveform is the same as the anode wire pitch,  $s = 2.82$  mm. These calculations show that the differential non-linearity passes through a very sharp minimum at  $f_x = 1.0$ . This is illustrated in Fig. 9, for the ratio  $w_r/h = 2.40$  and induced charge distribution with  $K_3 = 0.667$ . The theoretical minimum in DFNL in this case is extremely small, less than 0.2%, but such a value cannot be expected in reality. The most significant difference between the theoretical model and a real cathode is that a finite gap must exist between adjacent strip boundaries; this has been assumed to be zero in the model. In addition the apex of the zigzag pattern, as generated on the printed circuit board, has a finite radius of curvature, approximately 40 microns at present. It is assumed to be a perfect point in the model.

Figure 8 also shows that the anode wires pass over the zigzag shaped boundaries midway between the apices. If the anode wires are displaced from this optimum position, the model predicts that differential non-linearity increases, and that there

Table 1. Predicted and Measured Differential Nonlinearities  
 $C_1/C_2 = 3.80$ ;  $K_3 = 0.667$

$C_1'/C_1$	DFNL(%) Delay line	DFNL(%) Geometrical	DFNL(%) Exper- imental	$f_q$
1.00	19.5	19.9	20	0.14
2.54	2.80	2.99	4	0.14
5.31	11.2	9.97	10	0.14
10.2	15.3	14.5	16	0.14

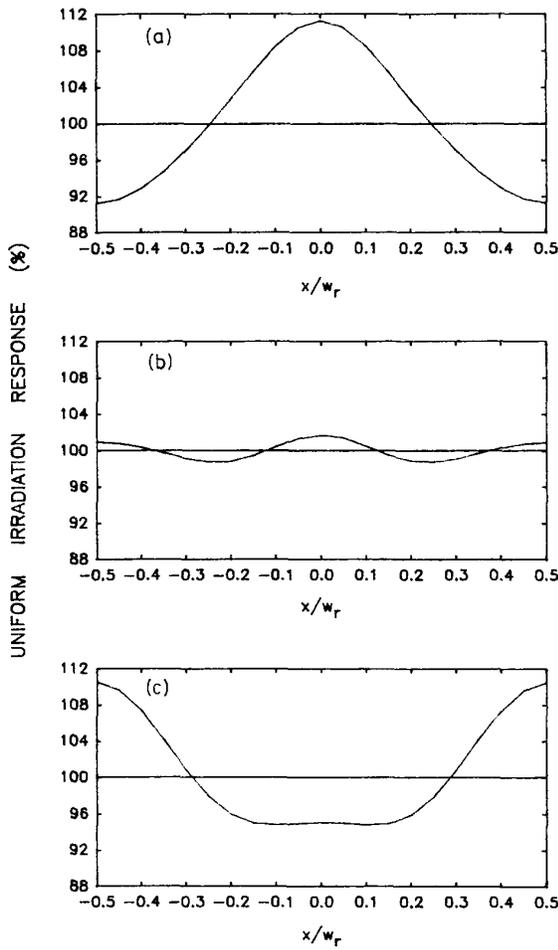


Fig. 6 Theoretical uniform irradiation responses for TIS cathode, with  $w_p/h = 2.40$ ,  $C_1/C_2 = 3.80$  and  $K_3 = 0.667$ .  
 (a)  $C_1'/C_1 = 1.00$ , DFNL = 19.5%;  
 (b)  $C_1'/C_1 = 2.54$ , DFNL = 2.99%;  
 (c)  $C_1'/C_1 = 10.2$ , DFNL = 14.5%.

is a constant offset in the output position signal. The magnitude of both the non-linearity and the offset is greatest when the anode wires pass over the apices of the zigzag; the variation of DFNL as a function of  $f_x$ , for this situation, is shown by the dashed line in Fig. 9.

Figure 10 shows examples of the uniform irradiation response obtained with ZZS cathodes, with  $w_p/h = 2.40$ , for three values of  $f_x$ . The DFNL decreases from 17% to about 7% as  $f_x$  changes from 0.9 to 1.0; as  $f_x$  changes further to 1.1 the DFNL remains at about 7% but the modulation changes in phase. Thus it is possible that a shallow minimum in DFNL would be observed experimentally near  $f_x = 1.05$ , rather than at  $f_x = 1.00$  as predicted in Fig. 9.

Comparing Figs. 9 and 10 it is clear that experimentally measured DFNL's are considerably larger than those predicted by the theoretical model. As noted above this could, at least in part, be due to the finite gap between adjacent strip boundaries. In fact, measurements on an earlier ZZS cathode with  $w_p/h = 2.0$  and  $f_x$  constant at unity, showed that the DFNL significantly

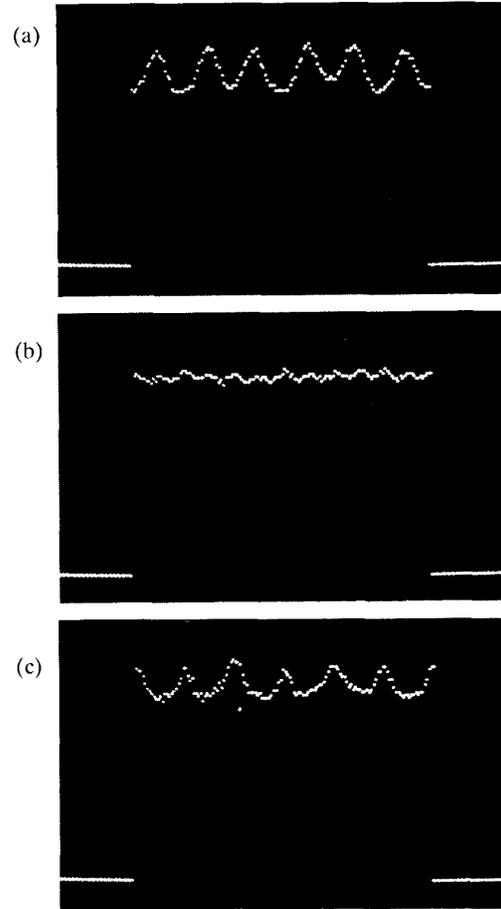


Fig. 7 Experimental uniform irradiation responses for TIS cathode, with  $w_p/h = 2.40$ .  
 (a)  $C_1'/C_1 = 1.0$ , DFNL  $\approx 20\%$ ;  
 (b) estimated  $C_1'/C_1 = 2.5$ , DFNL  $\approx 4\%$ ;  
 (c) estimated  $C_1'/C_1 = 10$ , DFNL  $\approx 16\%$ .

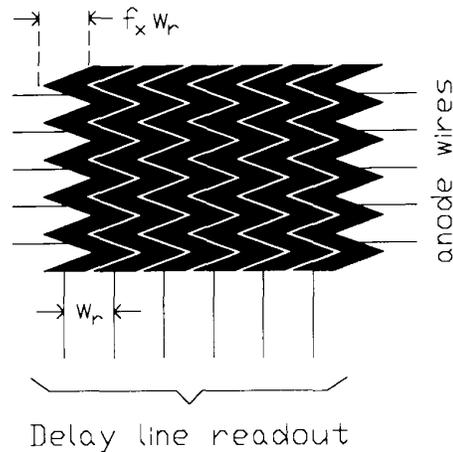


Fig. 8 Schematic of zigzag strip (ZZS) cathode and connections to delay line. The peak-to-peak depth of the zigzag boundary is  $f_x w_r$ ; the period is equal to the anode wire pitch,  $s$ . (In this diagram  $f_x = 1.0$  and  $s = 2.82$  mm).

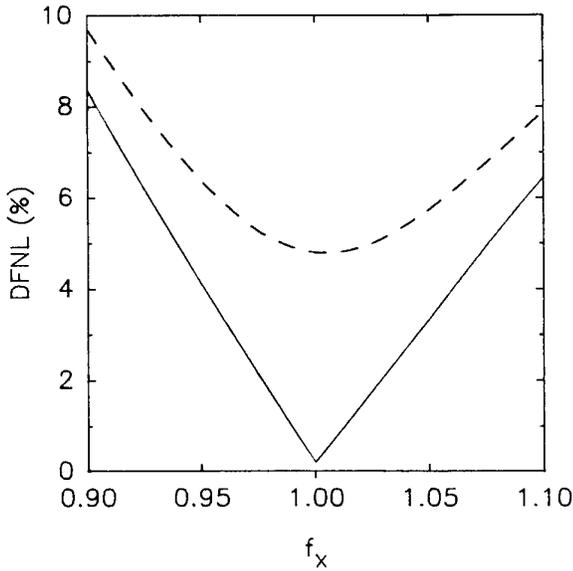


Fig. 9 Calculated differential non-linearity of zigzag strip (ZZS) cathode as a function of  $f_x$  (for  $w_r/h = 2.40$  and  $K_3 = 0.667$ ). The dashed curve shows the differential non-linearity when the anode wires pass over the apices of the zigzag boundaries.

increased when the gap to strip pitch ratio increased from 0.09 to 0.34. For the results in Fig. 10 the gap to strip pitch ratio was 0.09, about the smallest that can be realised with standard printed circuit board techniques with  $w_r = 3.81$  mm.

### 5. Discussion

Both the TIS and ZZS cathodes are designed to improve DFNL without any reduction in  $w_r$ , i.e. without increasing the number of readout nodes. It has been mentioned that this is particularly important for lumped parameter delay line readout, and in general it is always desirable to reduce the number of readout channels in any position encoding technique. In order to exaggerate the effect of increasing the sampling width, the present measurements were made, purposely, with the very large value  $w_r/h = 2.4$ . This resulted in a DFNL of 72% for conventional strip readout. This modulation is caused by a systematic error in encoded position which, in the present case, reaches a maximum of about 6% of  $w_r$  (or 0.23 mm) at 1/4 and 3/4 of the way across each strip.

For the TIS cathode it has been shown that, with careful optimisation of the coupling capacitor  $C_1'$ , it is possible to reduce the DFNL to about 4%. The quite good agreement in DFNL between experiment and theory for various values of  $C_1'$  should allow optimised values of  $C_1'$  to be chosen realistically for detector geometries different from the present one.

The TIS cathode has the special advantage of small inter-node capacitance, which lies between  $C_1/2$  and  $C_1/3$ . (By comparison, the inter-node capacitance on a conventional cathode would be at least  $C_1$ .) This makes the TIS scheme particularly suitable for position encoding by delay line, since unwanted signal dispersion reduces with decreasing inter-node capacitance.

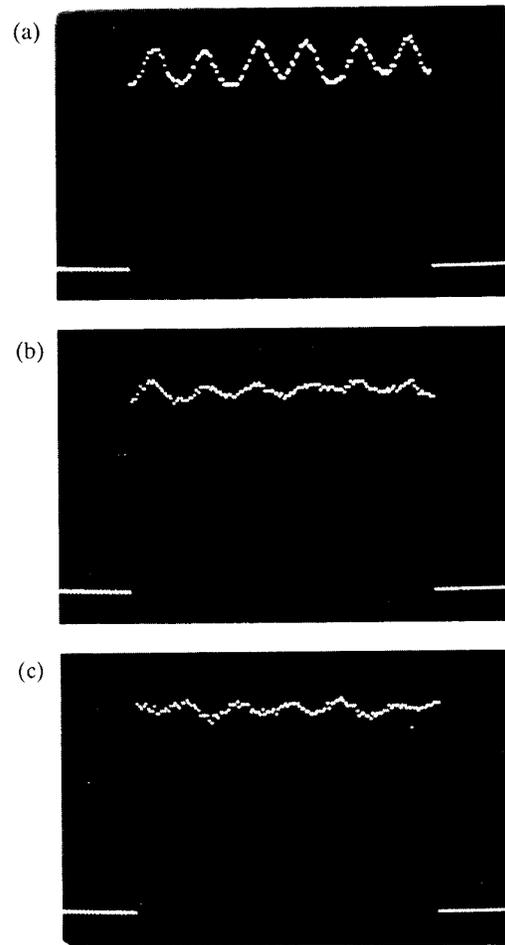


Fig. 10 Experimental uniform irradiation responses for ZZS cathode, with  $w_r/h = 2.40$ .

- (a)  $f_x = 0.9$ , DFNL  $\approx 17\%$ ;
- (b)  $f_x = 1.0$ , DFNL  $\approx 7\%$ ;
- (c)  $f_x = 1.1$ , DFNL  $\approx 7\%$ .

In comparing experimental results with predictions of the model, the most difficult problem was measuring an accurate value for  $C_2$  which is not only very small but also influenced significantly by dielectric at the edges of the cathode.

In the study of zigzag strips, or pads, for the TOPAZ TPC [10], there does not appear to have been a quantitative determination of the relationship between DFNL and zigzag depth,  $f_x w_r$ . In the present work we have developed such a theoretical model. While agreement between experiment and prediction is not close, the experimental results do follow approximately the predicted trend. The best experimental DFNL,  $\approx 7\%$ , is not very much worse than that for the optimised TIS cathode. An unfortunate property of the ZZS cathode is its relatively large interstrip capacitance, due in large part to the increase in boundary length between strips. For example, with  $w_r/h = 2.4$ , the boundary length of strips on the ZZS cathode is 2.9 times as long as on conventional strips. However, like DFNL, the exact interstrip capacitance is fairly dependent upon gap width. An advantage that the ZZS cathode has over the TIS cathode is its somewhat simpler construction.

The present theoretical predictions were made assuming an induced charge distribution determined by the single-parameter Gatti empirical formula [7,8], with  $K_3 = 0.667$ . In fact, somewhat surprisingly, the output position signal has been found to be rather insensitive to the precise shape of the distribution. More realistic distributions, corresponding to avalanche centroid angular positions  $+\pi/2$  and  $-\pi/2$ , with rms spread 40 degrees and at a time  $0.1 \mu\text{s}$ , have been determined by the method of Ref. [11]. The predicted difference between the position signals for these two distributions has a maximum value of about 2.6 microns (TIS cathode) or 0.3 microns (ZZS cathode). Thus, under the present experimental conditions, angular localisation should have no significant effect on resolution, nor should it produce, in the case of the ZZS cathode, any significant crosstalk. These predictions have been essentially confirmed experimentally.

In conclusion, we have shown that at the very unfavourably large value  $w_p/h = 2.4$ , where conventional strip geometry yielded a DFNL of 72%, the use of optimised TIS and ZZS cathodes can nevertheless produce relatively low values of DFNL, about 4% and 7% respectively in the present experimental arrangement. These results suggest that, if a more reasonable value for  $w_p/h$ , say 2.0, was used in a practical detector, then an acceptable DFNL figure would be obtained; compared with a conventional strip cathode, the number of readout channels would then be reduced by the valuable factor 2.5.

#### Acknowledgements

The major part of the detector construction was expertly carried out by Gene Von Achen. We acknowledge the assistance of Tony De Libero, Bob Di Nardo and Bo Yu in various phases of cathode board production. Helpful advice from Veljko Radeka and Joe Fischer is much appreciated.

#### References

- [1] E. Gatti, A. Longoni, H. Okuno, and P. Semenza, Nucl. Instr. and Meth. 163, 83 (1979).
- [2] V. Radeka and R. A. Boie, Nucl. Instr. and Meth. 178, 543 (1980).
- [3] E. Gatti, A. Longoni, and V. Radeka, Nucl. Instr. and Meth. in Phys. Res., 220, 445 (1984).
- [4] J. L. Alberi, IEEE Trans. Nucl. Sci., NS-24, 188 (1977).
- [5] G. C. Smith, J. Fischer, and V. Radeka, IEEE Trans. Nucl. Sci., NS-35, 409 (1988).
- [6] R. A. Boie, J. Fischer, Y. Inagaki, F. C. Merritt, V. Radeka, L. C. Rogers, and D. M. Xi, Nucl. Instr. and Meth., 201, 93 (1980).
- [7] E. Mathieson and J. S. Gordon, Nucl. Instr. and Meth. in Phys. Res., 227, 277 (1984).
- [8] E. Mathieson, Nucl. Instr. and Meth. in Phys. Res., A270, 602 (1988).
- [9] J. S. Gordon, E. Mathieson, and G. C. Smith, IEEE Trans. Nucl. Sci., NS-30, 342 (1983).
- [10] T. Miki, R. Itoh, and T. Kamae, Nucl. Instr. and Meth. in Phys. Res., A236, 64 (1985).
- [11] J. R. Thompson, J. S. Gordon, and E. Mathieson, Nucl. Instr. and Meth. in Phys. Res., A234, 505 (1985).