Precise Measurements of Small Linear and Angular Displacements with Capacitance Methods

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- Motivation: EDM G-2 Upgrade (by Gerald Bennet)
- Types of Displacement and Angle Sensors
- Advantages of Capacitive Displacement Sensors
- Readout Methods
- Limits to Sensitivity: Electrical and Mechanical Noise
- MEMS/Microelectronics Applications: Accelerometers

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P. Rehak
Types of Displacement and Angle Sensors

1: Interferometric Sensor

- interferometric technique: optical wavelength-scale resolution
- Measures relative movements: laser light must be on all the time unless the reference point is lost
Types of Displacement and Angle Sensors

2: Interpolating Sensors (e.g SONY MAGNESCALE)

- Uses spatial averaging: reduced differential non-linearity
- Large Dynamic range
- Integral non-linearity depends on accumulated error in N-S magnetization locations
Types of Displacement and Angle Sensors (cont)

LVDT: Linear Variable Differential Transformer

A ferromagnetic core moves with respect to two opposite windings, changing the coupled flux.

- S-shaped characteristics
- Bulk
- Slow response (uses low frequency)
- Sensitive to magnetic fields
- It is the most widely used displacement sensor
Types of Displacement and Angle Sensors (cont)

CAPACITIVE SENSORS

Piston type variable Capacitor

Differential Capacitors

![Diagram of piston type variable Capacitor]

![Diagram of differential Capacitors]

![Graph of output vs displacement]

(NOISE < 100μV RMS)
(BANDWIDTH ~ 3 KHz)
Advantages of Capacitive Sensors

Vary A => better linearity

\[ C = \varepsilon \cdot \frac{A}{d} \]

Vary d => better sensitivity over small displacement

- Excellent linearity over entire dynamic range when Area is changed (since stray electric fields are small)
- The system responds to average displacement of a large area of a moving electrode
- Freedom of electrode materials and geometry for demanding environments and applications
- Fractional change in capacitance can be made large
- Capacitive sensors can be made to respond to displacements in one direction only
- The forces exerted by the measuring apparatus are electrostatic, and usually small enough so that they can be disregarded
- Capacitors are noiseless: excellent S/N ratio can be obtained (or their dissipation factor D is large enough that the dominant noise sources are elsewhere)
Types of Readout

Two major categories:
1. Readout based on Resonance
2. Readout based on Bridge method

Readout based on Resonance

1. Measure frequency change of an oscillator built around the variable capacitor
2. Excite at resonance, measure amplitude change
3. Excite at resonance, measure phase change
4. Use feedback loop and VCO oscillator to track resonance change
Readout based on Resonance (cont)

Advantages

• Makes use of the high Q of a resonant circuit: does not require a low noise preamplifier
• Sensitive
• Simple and straightforward

Disadvantages

• All resonating elements are created equal: it cannot distinguish a change in L from a change in C.
  The overall stability depends on the stability of BOTH L and C, with different temperature coefficients and stray effects.
• Cannot take advantage of differential capacitance change
Readout Based on Resonance: Example
Rocking Beam Balancing in Atomic Force Microscope (G. L. Miller)
Types of Readout: AC Bridge

\[ C_1 - C_2 = \varepsilon \frac{A}{(\delta x - x_0)} - \varepsilon \frac{A}{(\delta x + x_0)} = 2 C_0 \frac{\delta x}{x_0} \]

\( A = 400 \text{ mm}^2 \quad x_0 = 75 \mu \text{m} \quad C_1 = C_2 = 50 \text{ pF} \)

\( l = 30 \text{ mm} \quad \delta \theta = 1 \text{ nrad} \quad \delta x = \delta \theta l = 30 \times 10^{-12} \text{ m} \)

\( \delta C = C_1 - C_2 = 40 \text{ aF} \)

\[ F = 0 \text{ for a symmetric system} \]

\[ \text{Force} = \frac{1}{2} \frac{C_1}{x_0} v^2 \]
Types of Readout: AC Bridge

Readout Equivalent Circuit

For zero output from the capacitance balance detector must be:

$$\frac{C_1}{C_2} = 1 + L_B(C_2 + C_B)\omega^2 - L_A(C_1 + C_A)\omega^2 + \frac{R_E}{R_A} - \frac{R_F}{R_B} + \frac{\delta V_1}{V_1}$$

<table>
<thead>
<tr>
<th>(L_A, L_B)</th>
<th>0.4 mH</th>
<th>ratio transformer Leakage inductance</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_E, R_F)</td>
<td>&lt;1 (\Omega)</td>
<td>transformer secondary DC resistance</td>
</tr>
<tr>
<td>(R_A, R_B)</td>
<td>&gt;10 M(\Omega)</td>
<td>Dielectric losses</td>
</tr>
<tr>
<td>(C_A, C_B)</td>
<td>~ 1000 pF</td>
<td>Stray capacitance to “ground” (cables, interwinding C)</td>
</tr>
<tr>
<td>(R_D, C_D)</td>
<td></td>
<td>Impedance shunting the preamplifier. Does not affect balance</td>
</tr>
<tr>
<td>(Z_D)</td>
<td>low</td>
<td>Current preamplifier input impedance</td>
</tr>
</tbody>
</table>
AC Bridge: Noise

The “equivalent noise capacitance” can be calculated as:

$$\bar{C}_n^2 = \frac{1}{V_1^2} \left( \frac{i_n^2}{\omega_0^2 C_T^2} + \frac{e_n^2}{\omega_0^2 C_T^2} \right) BW$$
AC Bridge: Noise (cont)

To reduce noise:

- Increase $V_1$
- Decrease Bandwidth (i.e. increase averaging time)
- Decrease $C_T$ (depends mostly on connection length, strays etc.)

Example: Tiltmeter

$$
\begin{align*}
C_T &= 1 \text{ nF} \\
e_n &= 1 \text{ nV/Hz}^{1/2} \\
V_1 &= 3 \text{ V}
\end{align*}
\}

\left\{ \begin{array}{l}
C_n = 4 \times 10^{-19} \text{ F} = 400 \text{ zF} \\
C_n = 2 \times 10^{-21} \text{ F} = 2 \text{ zF}
\end{array} \right.

\text{Minimum Signal (for 1 nrad angular displacement) = 40 aF = 20 } \times 10^{-18} \text{ F} \Rightarrow S/N=100

Example: Monolithic MEMS circuit

$$
\begin{align*}
C_T &= 1 \text{ pF} \\
e_n &= 10 \text{ nV/Hz}^{1/2} \\
V_1 &= 5 \text{ V}
\end{align*}
\}

\left\{ \begin{array}{l}
C_n = 4 \times 10^{-19} \text{ F} = 400 \text{ zF} \\
C_n = 2 \times 10^{-21} \text{ F} = 2 \text{ zF}
\end{array} \right.$
Mechanical Noise

The mechanical rms fluctuation can be computed by means of the fluctuation dissipation theorem. The rms fluctuation of the displacement of a suspended mass m is:

\[
\delta x^2 \approx \frac{k_B T}{m \omega_0^2 \tau S}
\]

Where:

\[k_B = 1.38 \times 10^{23} \text{ J/K}\]
\[m \sim 20 \text{ g}\]
\[\omega_0 = \text{mechanical resonant frequency} \sim 10^5 \text{ rad/sec}\]
\[\tau = \text{damping time constant} \sim 5-10 \text{ sec}\]
\[S = \text{Averaging period} \sim 1 \text{ sec to minutes}\]

So that \((\delta x^2)^{(1/2)} \sim 10^{-17} \text{ m}\) giving a “noise” too small to be detected.

Absolute Limits:

Since the averaging S and damping time constant \(\tau\) could be increased (e.g. mounting the tiltmeter in vacuum to reduce friction and reducing the mechanical losses of the hinge) there is no fundamental limit to the mechanically generated noise.
Other Sources of Errors

- Temperature variations
  
  Thermal expansion => cancelled in a symmetrical design
  
  Effects on the readout electronics: Gain Variation
  
  temperature dependence of dielectric constant (2 $10^{-6}$/°C for dry air at STP;

  700 $10^{-6}$ for moist air

- Humidity variations
  
  at 20°C a change in humidity from 40 to 90% changes the dielectric constant by 200 ppm

- Pressure changes
  
  a pressure change of 1 atm at 20°C changes the dielectric constant by 200 ppm
  
  causes dimensional changes (a brass cube of 1 cm contracts by 3 µm for a 1 atm change)

- Oxidation of surfaces (rodium plating recommended)

- Stability of materials (70-30 brass gives good results)

- Creep of materials

- Relaxation of screw tension

- Microseismicity (about 2 µm peak to peak displacement, period 3-8 s)
MEMS Accelerometers

It consists of multiple fingers on each side of a movable center member.

They constitute the center plates of a paralleled set of differential capacitors. Pairs of fixed fingers attached to the substrate interleave with the beam fingers to form the outer capacitor plates. The beam is supported by tethers which serve as mechanical spring.

“Force” fingers are used for calibration

mass = 0.5 µg

SIZE: 0.5 mm x 0.4 mm, 2 µm thick,

- **Requirement:** Avoid “stiction” => rigid cantilevered beam
Sensor operation; ADI’s implementation

- Folded tethers have more consistent spring constants, leading to better part to part consistency
Self test operation

- Extra fixed outer plates may be added which when exited, force the proof mass to move. So you can electronically test the accelerometer.

Additional fixed outer plates are electrically excited to induce movement of the proof mass. Acceleration is measured by the standard fixed plates as usual.
Interesting facts

- 0.1µgrams Proof Mass
- 0.1pF per Side for the Differential Capacitor
- 20aF (10^{-18}f) Smallest Detectable Capacitance Change
- Total Capacitance Change for Full-scale is 10fF
- 1.3µm Gaps Between Capacitor Plates
- 0.2Å Minimum Detectable Beam Deflection (one tenth of an Atomic diameter)
- 1.6 μm Between the Suspended Beam and Substrate
- 10 to 22kHz Resonant Frequency of Beam
MEMS Accelerometers:

Readout

Noise

Specifications | ADXL105 | ADXL202 | ADXL05 | ADXL50
---|---|---|---|---
Range (g) | ±5 | ±2 | ±5 | ±50
Noise (µg/√Hz) | 225 | 500 | 500 | 6500
Bandwidth (kHz) | 10 | 5 | 5 | 6
Supply Current (mA) | 2 | 0.6 | 10 | 6
Number of Axes | 1 | 2 | 1 | 1
Output Type | Analog | Analog/Digital | Analog | Analog

ADXL105:
C = 150 fF
Sensitivity = 100 aF for 1g acceleration
noise = 225 g/Hz\(^{1/2}\)
“capacitance” noise = 22.5 \(10^{-21}\) F/Hz\(^{1/2}\) = 22.5 zF/Hz\(^{1/2}\)
Bibliography

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8. A good reference book on the art of winding of ratio transformers and high precision AC bridge techniques is the book ”Coaxial AC Bridges” by Brian Kibble and G. Rayner, published by Bristol A. Hilger, 1984. It is out of print, but a photocopy can be ordered through the UK National Physical Laboratory at the URL: http://www.npl.co.uk/electromagnetic/cem-publications/goodpractice.html