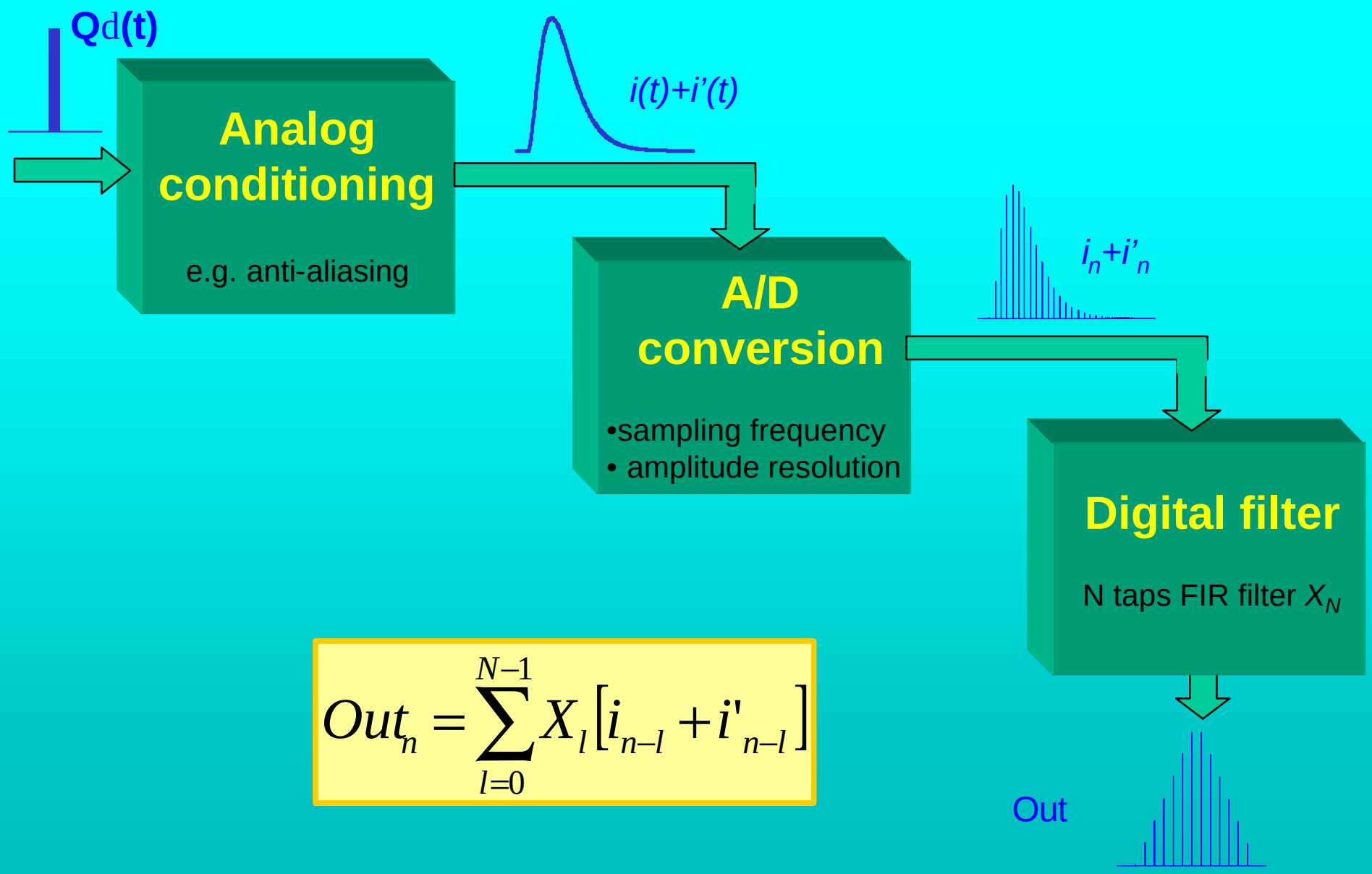
Politecnico di Milano
Department of Electronics and Information Science
Piazza Leonardo da Vinci 32 Milano MI 20133 ITALY

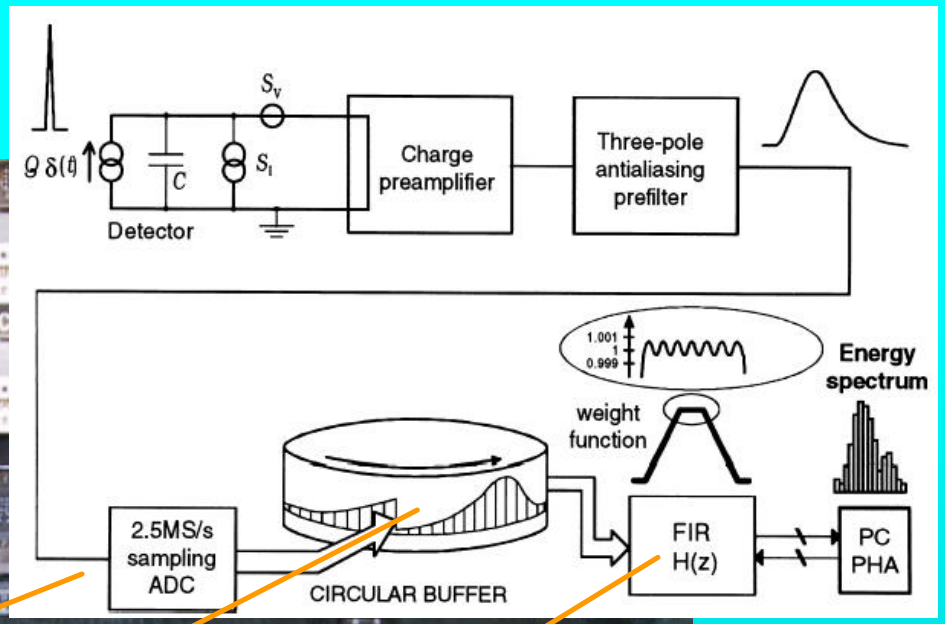
A new method for LMS synthesis of optimum FIR filters with arbitrary time and frequency constraints and noises

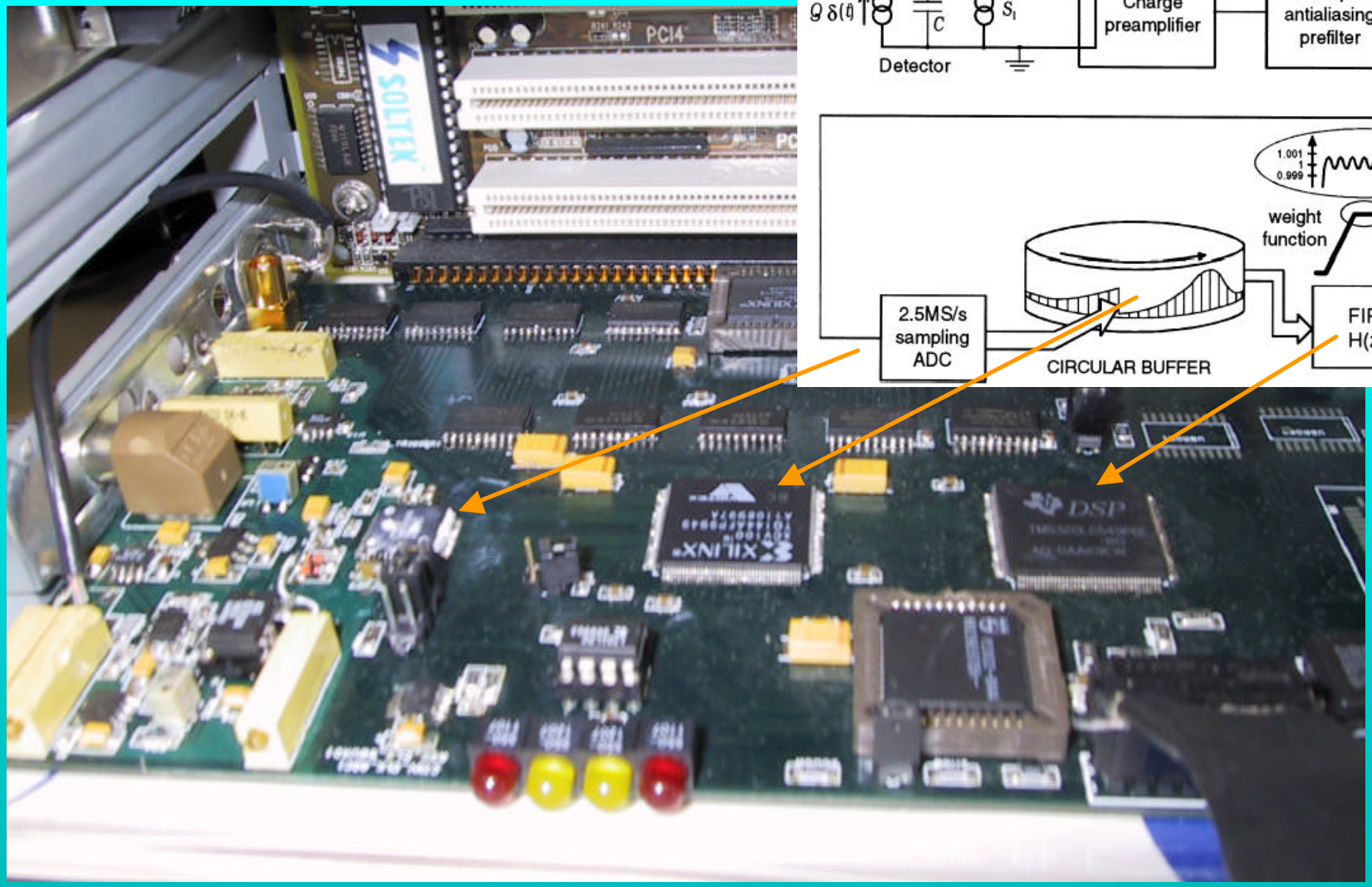
Emilio Gatti, Angelo Geraci, Giancarlo Ripamonti
Stefano Riboldi, Roberto Abbiati



$$Out_n = \sum_{l=0}^{N-1} X_l \left[i_{n-l} + i'_{n-l} \right]$$

Implementation example





Pulse response (*Out*) variance

$$\mathrm{var}(Out) = \begin{bmatrix} X_0 & X_1 & \cdot & \cdot & X_{N-1} \end{bmatrix} \begin{bmatrix} \overline{i'^2_0} & \overline{i'_0 i'_1} & \cdot & \cdot & \overline{i'_0 i'_{N-1}} \\ \overline{i'_1 i'_0} & \overline{i'^2_1} & \cdot & \cdot & \overline{i'_1 i'_{N-1}} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \overline{i'_{N-1} i'_0} & \overline{i'_{N-1} i'_1} & \cdot & \cdot & \overline{i'^2_{N-1}} \end{bmatrix} \begin{bmatrix} X_0 \\ X_1 \\ \cdot \\ \cdot \\ X_{N-1} \end{bmatrix}$$

Example: N=2

$$\mathbf{var}\ (OUT) = X_0^2 \overline{i'^2_0} + 2X_0 X_1 \overline{i'_0 i'_1} + X_1^2 \overline{i'^2_1}$$

Time constraint on the *k* value (V_k) of *Out*

$$\left[E(Out)_k - V_k\right]^2 = \left[\sum_{l=0}^{N-1} X_l \cdot i_{k-l} - V_k\right]^2$$

Any request on the *OUT* sequence domain
(i.e. **flat top**, **finite duration**, **slope**, etc.)
translates into punctual constraints and the corresponding quadratic forms are added together to build up an overall functional.

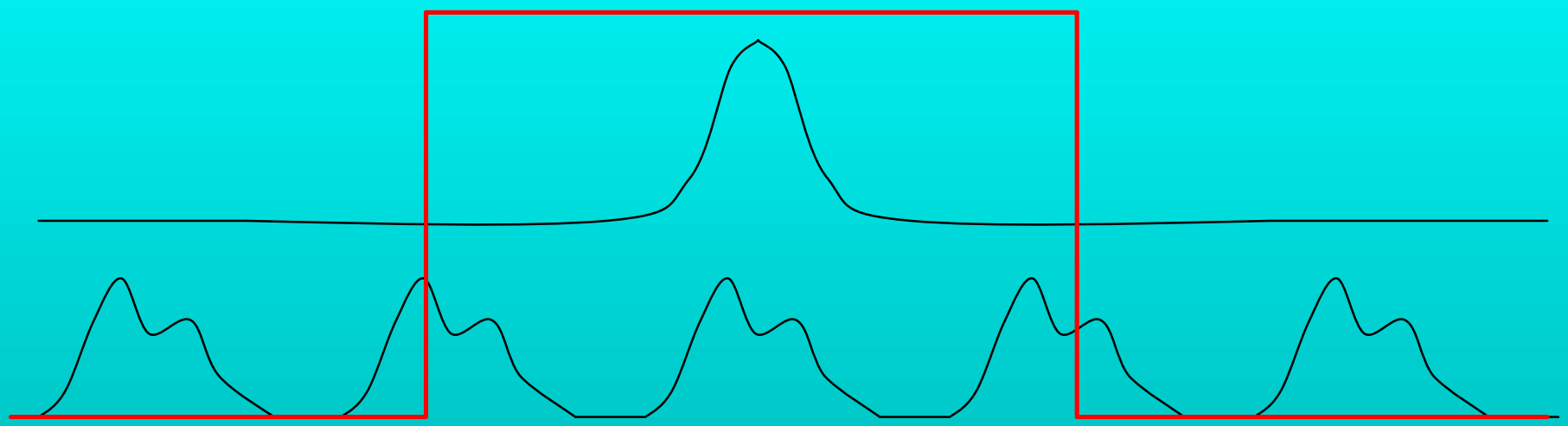
Frequency constraint for removal of f_0 frequency in *Out*

$$\left|\Im\{X\}\right|_{w_0} = \left|\sum_{l=0}^{N-1} X(l) \cdot e^{+jw_0 T_c l}\right| =$$

$$= \sqrt{\left(\sum_{l=0}^{N-1} X(l) \cdot \cos(w_0 T_c l)\right)^2 + \left(\sum_{l=0}^{N-1} X(l) \cdot sin(w_0 T_c l)\right)^2}$$

Periodic signal removal

$$\mathrm{var}(Out) = \begin{bmatrix} X_0 & X_1 & \cdot & \cdot & X_{N-1} \end{bmatrix} \begin{bmatrix} \overline{i'^2_0} & \overline{i'_0 i'_1} & \cdot & \cdot & \overline{i'_0 i'_{N-1}} \\ \overline{i'_1 i'_0} & \overline{i'^2_1} & \cdot & \cdot & \overline{i'_1 i'_{N-1}} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \overline{i'_{N-1} i'_0} & \overline{i'_{N-1} i'_1} & \cdot & \cdot & \overline{i'^2_{N-1}} \end{bmatrix} \begin{bmatrix} X_0 \\ X_1 \\ \cdot \\ \cdot \\ X_{N-1} \end{bmatrix}$$



$i'_i i'_k$ calculated in a finite time window

Tipical functional F to be minimized

$$F = P_1 \operatorname{var}(Out) + P_2 \left[E(Out_k) - 1 \right]^2 + \cdots + P_i \left[\cdots\cdot \right]^2$$

System of N equations to be solved

$$\frac{\partial F}{\partial X_0} = 0$$

$$\frac{\partial F}{\partial X_1} = 0$$

$$\cdot$$

$$\cdot$$

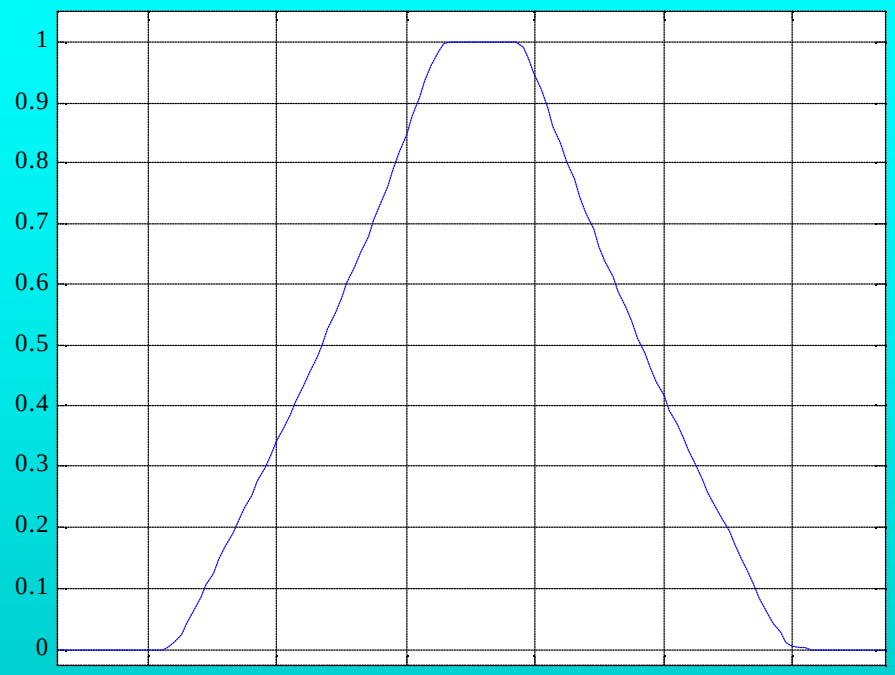
$$\frac{\partial F}{\partial X_{N-1}} = 0$$

$\{X_i\}$, i.e. OUT

N.B.

Arbitrary weights P_k

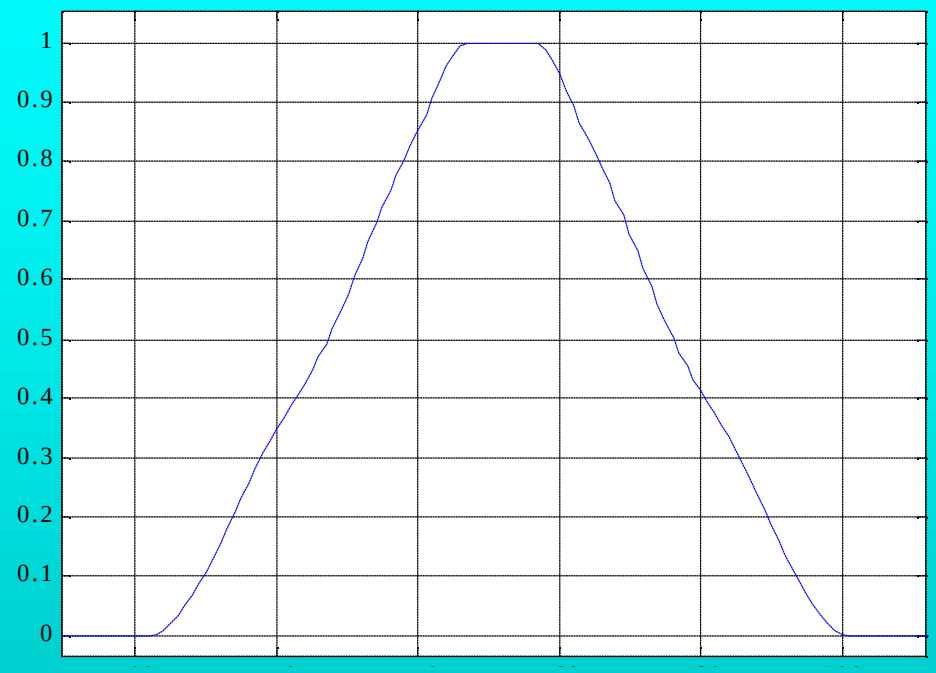
- **Optimum finite width pulse response with and without the <u>frequency constraint</u> of f_0=600kHz rejection**



0 **10**

Time [msec]

WITHOUT

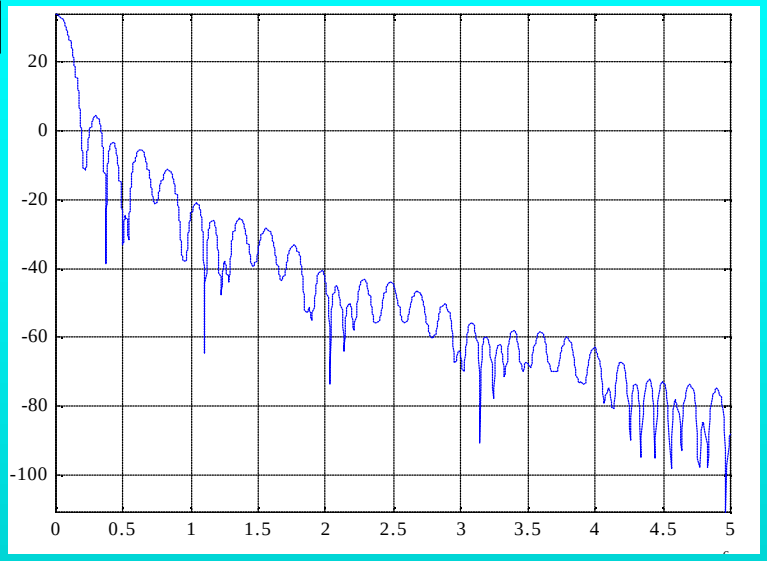


0 **10**

Time [msec]

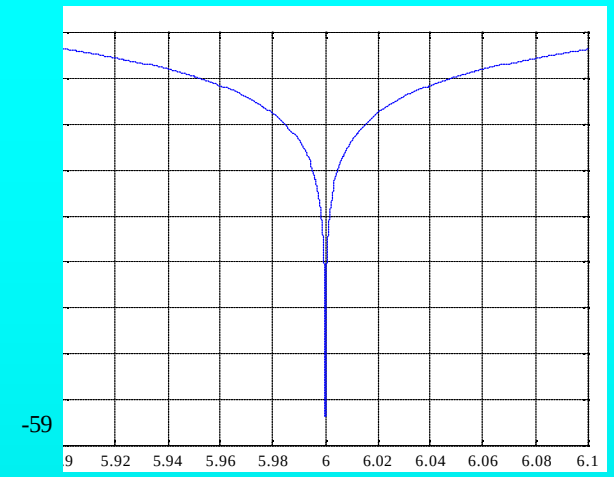
WITH

[dB]



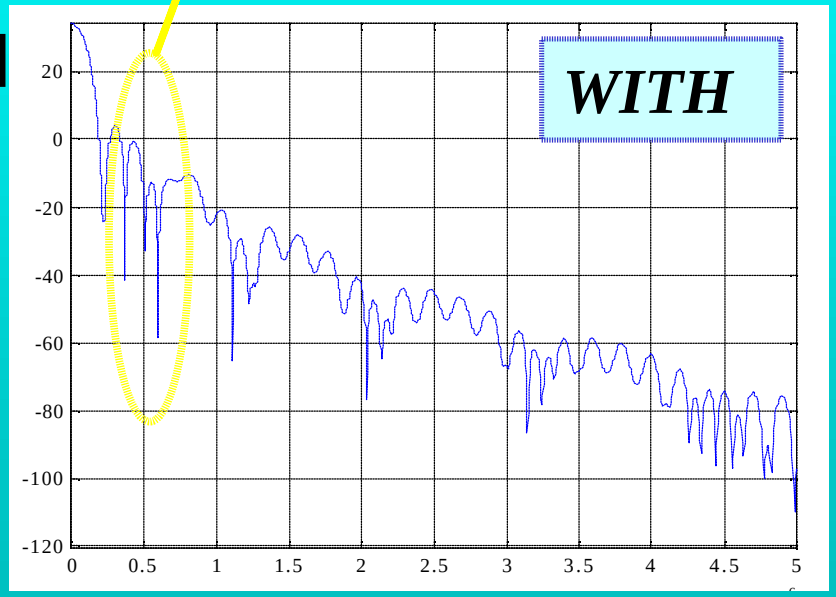
Frequency [MHz]

WITHOUT

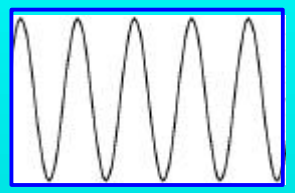


Frequency [x100kHz]

[dB]



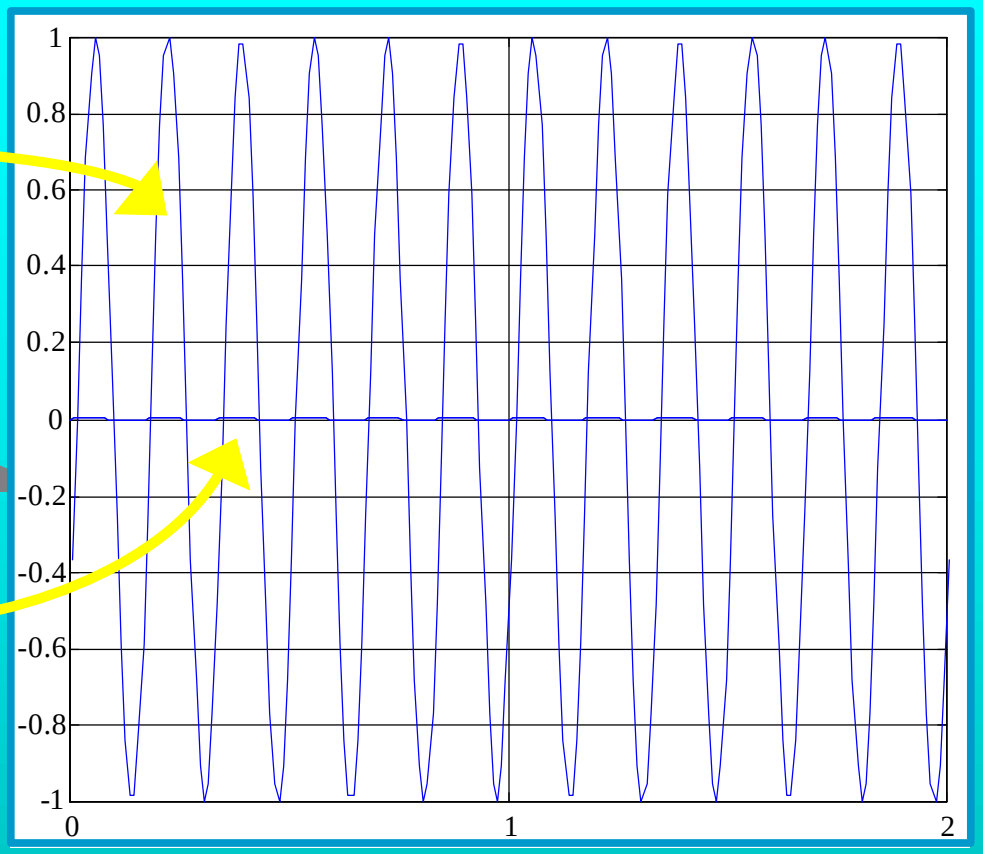
Frequency [MHz]



sin $(w_0\, t)$

FIR not filtering f_0

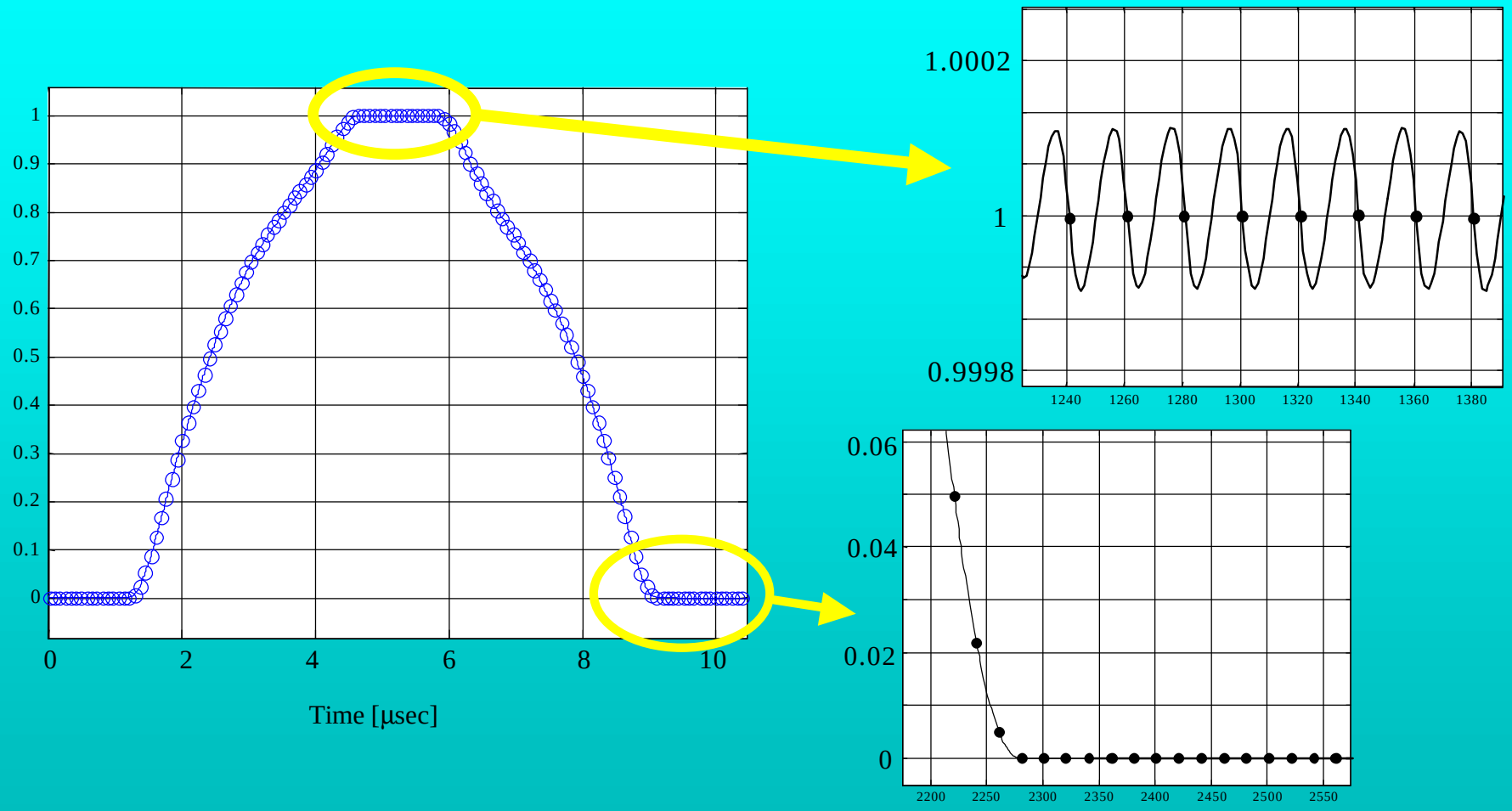
FIR Filtering f_0



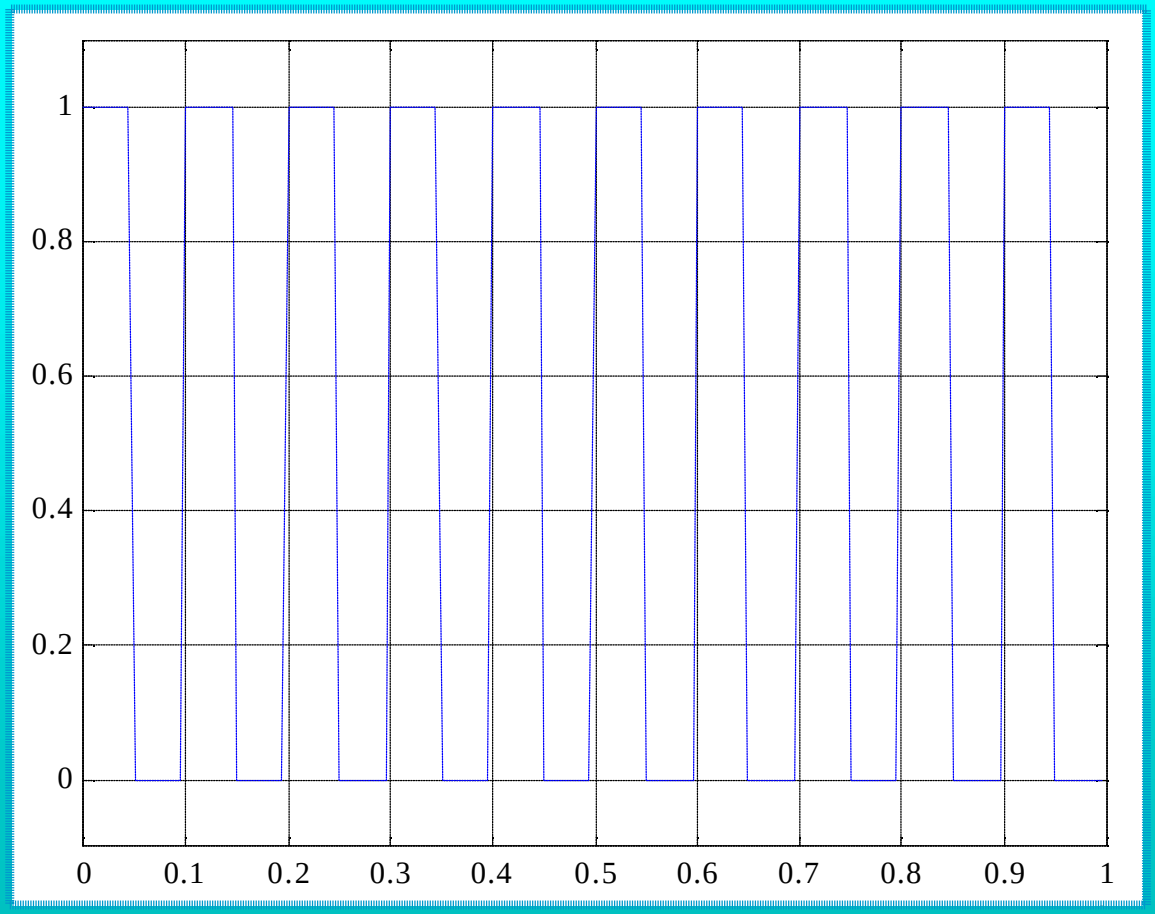
Time [x10 μsec]

practically a straight line of 0 amplitude (< 10^{-3})

- **Optimum finite width pulse response with <u>time constraints</u> of flat top and finite width**



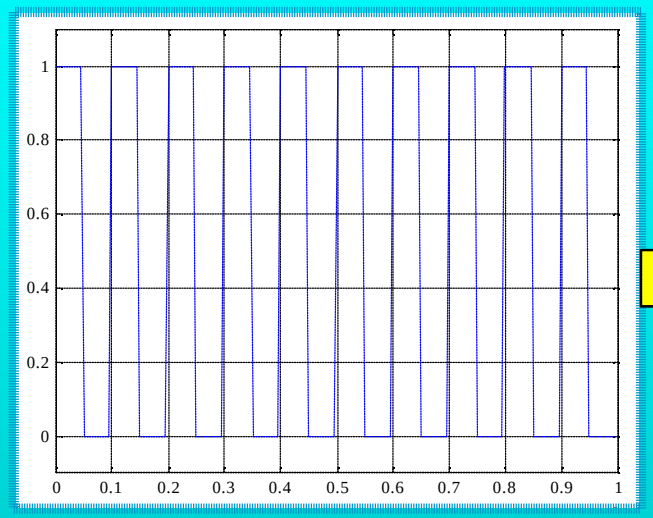
- **Optimum finite width pulse response with <u>disturbance</u> rejection (e.g. clock interference)**



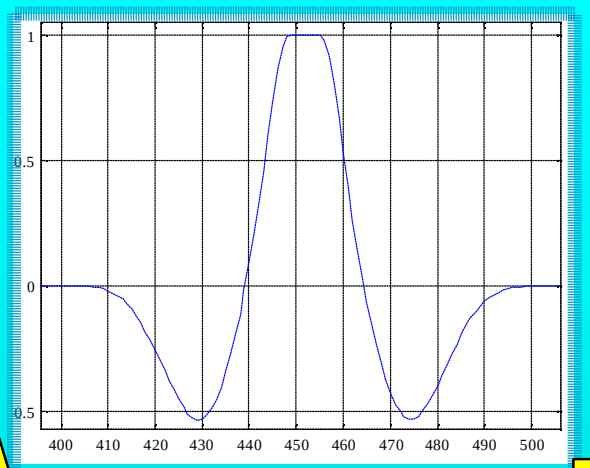
Time [x20 msec]

FIR filter

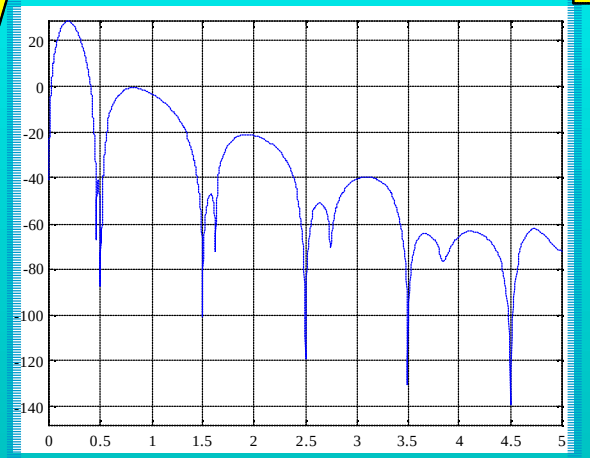
Input



Time [x20 msec]

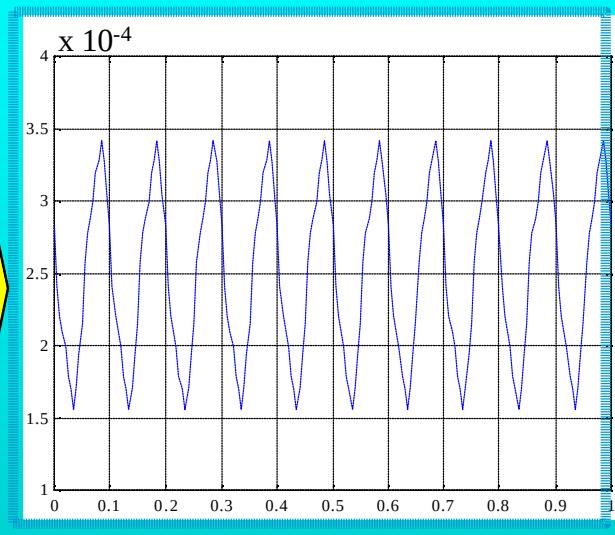


Time [samples]



Frequency [x20 MHz]

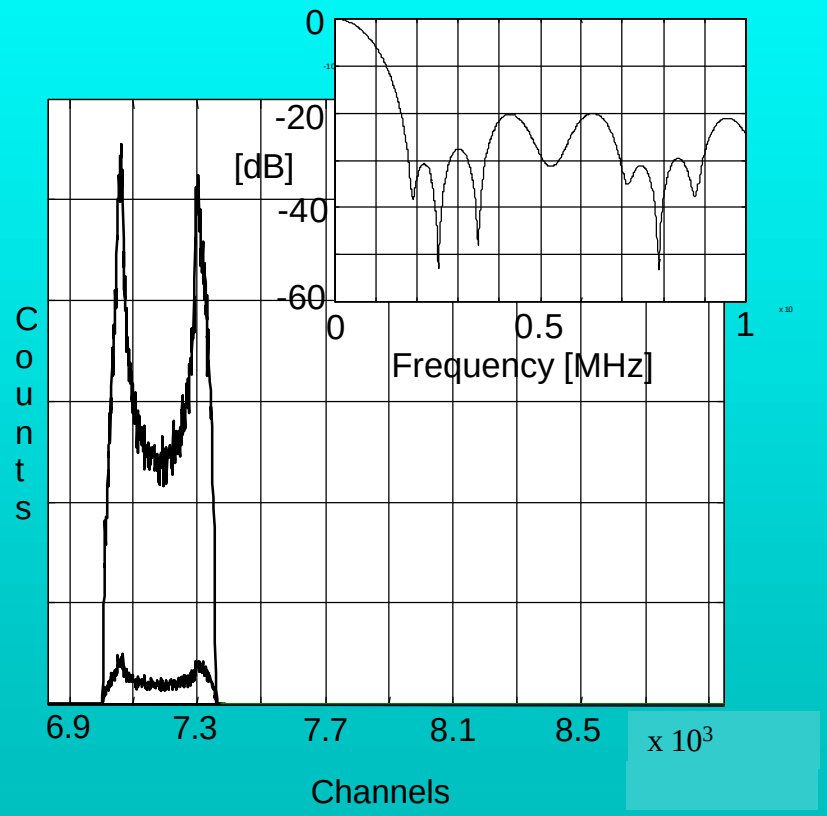
Output



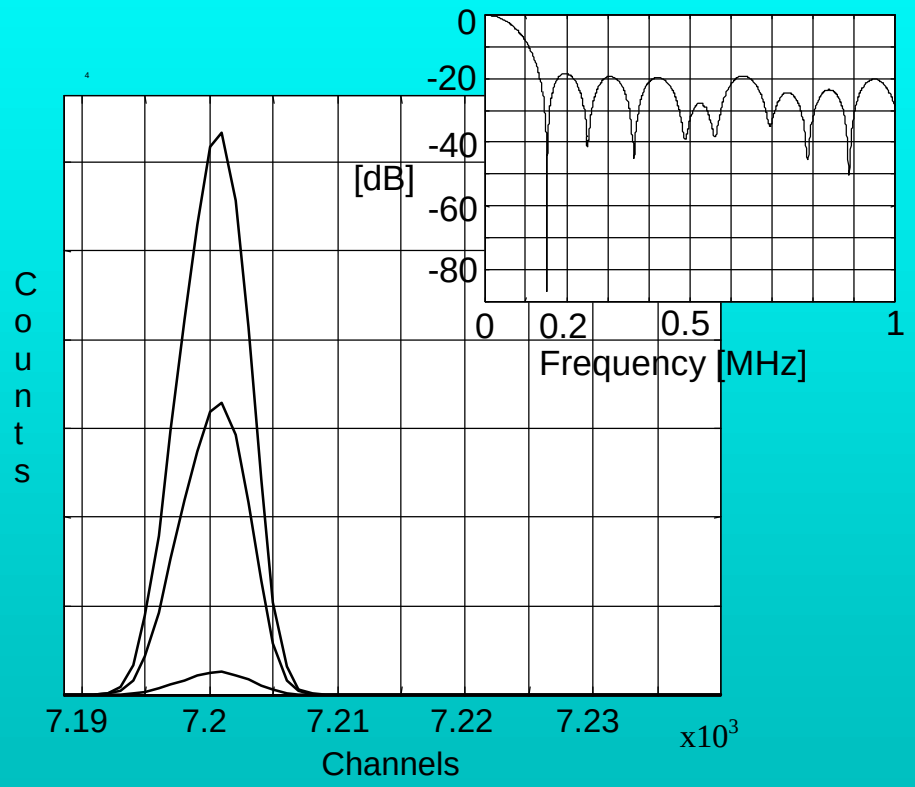
Time [x20 msec]

Experimental energy spectra with and without suppression of a 150kHz sinusoidal disturbance

WITHOUT



WITH



Conclusions

- **Fast** method to calculate optimum filters
- with **arbitrary (time and frequency) constraints** and **any noise** power spectrum
- **no burden** PC implementation
- **allows** to explore loss in ENC^2 due to constraints and taylor them for **best compromise** between precise **requirement** addressing and energy **resolution**