Detector Signal Processing

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Slide Captions and Notes:

1. Signal Formation and Ramo's Theorem

Fig. 4. This figure illustrates Ramo's theorem for induced signals, current and charge. $\boldsymbol{E_w}$ is the weighting field in units of 1/cm, and it is a measure of electrostatic coupling between the moving charge and the sensing electrode. The procedure to calculate the induced current as a function of time is as follows. First, the weighting field is determined by solving Poisson's equation analytically or numerically assuming unity potential on the sensing electrode of interest and zero potential on all other electrodes. Next, the velocity of the moving charge as a function of position is determined from the operating (applied) field on the detector. This gives the induced current as a function of the position of the moving charge. Third, the position of the moving charge as a function of time is determined by solving the equations of motion. This is necessary in the case of ballistic motion of charge, but it is simple in the case of transport by drift as the charge carriers follow the applied electric field.
If we are interested only in the total induced charge and not in the waveforms, the induced charge is simply given by the difference in the weighting potentials between any two positions of the moving charge. An example of the weighting-field (potential) profiles is illustrated by the plot of equipotential lines for planar geometry with a strip sensing electrode. The operating (applied field) in this case is uniform and perpendicular to the electrodes. The *weighting field map* is in general quite different from that of the *operating field*; the two field maps are identical only in some special cases.
The minus sign in Ramo's equation for the induced current results from the arbitrary assumption of induced current *into* the electrode being *positive*.

Fig. 5. This is a simple case where the real (operating) electric field and the weighting field have the same form $\boldsymbol{=1/d}$. The induced current waveforms shown are for a semiconductor detector with different electron and hole mobilities. For extended ionization the waveforms result from superposition of the waveforms for localized ionization, and the currents decrease as the carriers arrive at the electrodes from different initial positions within the bulk of the detector.

Fig. 6. The sketch in (a) shows conceptually how is the weighting field (potential) defined: the sensing electrode is connected to unity potential, and all other electrodes to zero potential. The equipotential lines in (b) illustrate the solution for this case., showing two strips next to the sensing electrode. A great variety of results for the induced current and charge may arise in an electrode structure, such as this, depending on the particle type detected (distribution of ionization) and on the ratio of the charge observation measurement or integration) time and the charge carrier transit time. The current waveforms shown are drawn qualitatively for a simple example. The operating field is

assumed uniform and perpendicular to the electrode planes. Charge q_m traversing the full distance between the electrodes along line 1 is observed as $Q_1 = -q_m$, while the current decreases with distance from the sensing electrode 1, as the electrostatic coupling decreases. For a charge moving along line 2, the induced charge (i.e., the difference between the weighting potentials) is zero, if the measurement time is longer than the transit time. For a short measurement time a net induced charge is observed. The induced current waveform (the "crosstalk signal") is bipolar, since the weighting field direction changes along the path.

Fig. 7. Operating principle of a proportional counter. Induced current as a function of the position of positive ions and time during their drift from the anode region to the cathode. Note that the signal of interest containing most of the timing information is produced while the positive ions have moved only a few radii toward the cathode.

Fig. 8. Coplanar grid readout. This type of readout is commonly used with Cadmium Zinc Telluride (CZT) detectors. In such materials only electrons are collected (from the ionization produced by gamma rays or x-rays), the holes suffering from very low mobility and trapping. With planar parallel plane electrodes the induced charge for single carrier collection is dependent on the position (depth) where the ionization took place. In the coplanar grid concept one set of alternate strips is biased slightly more positively with respect to the other set of strips. This results in a drift field such that the signal electrons are collected on one set of strips only. The weighting potential (field) for both sets of strips is identical. The induced charges and currents are quite different. Their respective differences are independent of the position, as can be concluded by following the weighting potential plot from any point on the planar sloped part of the plot to unity weighting potential for the collecting electrode, and (across the saddle) to zero potential for the noncollecting electrode.

Fig. 9. Weighting field (potential) for strip and pixel electrode configuration. The shielding effect is proportional to the ratio of the distance between the planar electrodes and the strip width (i.e., pixel radius). The shielding effect is more pronounced for pixels than for strips. The result of these configurations is that the signal charge (integral of the induced current) is independent of the position of the origin of ionization for most of the volume of the detector except near the readout electrodes. This effect is used in detectors where only electrons are collected during the integration time, such as Cadmium Zinc Telluride (CZT), and some gas and noble liquid detectors. To illustrate this, histograms are shown for a strip and pixel illuminated by a beam of penetrating x-rays absorbed uniformly through the detector.

2 .Noise Mechanisms

Fig. 11. The basis of a noise process can be represented as a sequence of randomly generated elementary impulses that has a Poisson distribution in time and mean rate of occurrence $\langle n \rangle$. Upon acting on a physical system with impulse response much longer than $\langle n \rangle^{-1}$, the characteristic noise waveforms (e.g., such as those we observe on an

oscilloscope) are produced as a superposition of responses to individual impulses. The noise variance at the output of the physical system (a simple RC filter or a complete readout system) is calculated by using Campbell's theorem, which states that the sum of mean square contributions of all preceding impulses equals the variance. The expressions for the variance are given after subtracting the mean value. The variance is determined by the rate of impulses $\langle n \rangle$, their area q (charge), and by the impulse response (i.e., the weighting function $w(t)$ of the measurement system, the preamplifier and the subsequent readout chain).

For detailed description: V.R., Ann. Rev. Nucl. Part. Sci. **38** (1988) 217.

Fig. 12. Almost any noise spectrum can be generated from a random sequence of impulses (i.e., white noise with "infinite bandwidth") by using an appropriate filter. "Infinite bandwidth" implies a noise spectrum which is flat over the frequency range where our measurement system has a non-zero response. Simple integration of white noise results in "random walk" with $1/f^2$ spectrum. An elementary impulse response for generation of this noise is the step function $U(t)$. Generation of $1/|f|$ noise is somewhat more elaborate. It requires fractional integration of *order one half*. The impulse response of the transforming filter is $U(t)/t^{1/2}$, as shown in the figure. The basic feature of any noise generating mechanism for low frequency divergent noises is an "infinitely long memory", i.e., very long memory, for individual independent elementary perturbations. For a discussion of the basics: [Ref. 10].

Fig. 13. A noise spectrum very close to $1/|f|$ can be generated by superposition of relaxation processes with uniform distribution of life times. The relaxation process is described by the probability $U(t)\ exp(-t/\tau)$, which represents a step change with exponential decay.

Trapping-detrapping in semiconductors is one such possible mechanism for generation of $1/|f|$ noise. A single trap in the gate oxide (or the interface between the gate oxide and the channel) of an MOS transistor results in a drain current waveform shown in the upper left of the figure. This phenomenon is referred to as the ***random telegraph noise*** [Refs.43,44] and it is observed readily in the very small transistors in deep submicron technology. The spectral density of such a noise is Lorentzian as shown in the figure. Superposition of the spectra for a number of traps with a distribution of life times results in $1/|f|$ noise.

A hardware generator of $1/|f|$ noise can be made using the fact that white noise passed through a simple RC integrator has a Lorentzian spectral density. A circuit which transforms white noise into $1/|f|$ noise can be made requiring about one time constant (one RC circuit) per decade of frequency, as shown in [Ref. 10].

Fig. 14. $1/|f|$ noise is one of the fractal processes, and its waveform preserves the same features independently of the time scale. Another expression of this is independence of the measurement variance (i.e., the noise power) upon the time scale of the measurement as long as the ratio of the high frequency and the low frequency cutoffs remains constant. As the bandpass moves along the frequency spectrum the spectral density integral (i.e.,

the measurement variance) remains constant for f_h/f_l=***const.*** In detector pulse processing it is well known that the contribution of ***1/|f|*** noise to the equivalent noise charge (ENC) remains independent of the shaping time.

3. Equivalent Noise Charge (ENC) Calculation

Fig. 16. This figure illustrates principal noise sources in charge amplifiers and an equivalent diagram for calculation of the equivalent noise charge (ENC).

Two elementary noise generators are included in the equivalent circuit, a series noise voltage generator representing the noise in the amplifying device, and a parallel noise current generator representing various noise sources not inherent to amplification (detector leakage current noise, parallel resistor noise, etc.). Both types of noise are assumed to have a white spectrum. Two forms of presentation of these two noise sources in terms of a sequence of random pulses are shown, as ***charge (or voltage)*** at the input of the amplifier, and as a ***current*** injected into the input capacitance (comprised of the detector+amplifier+parasitic capacitances). *The presentation of the series noise in terms of a current into the detector input is the derivative of the charge (voltage) representation.* The sequence of voltage impulses representing the amplifier series noise thus corresponds to an equivalent sequence of ***current doublets*** (derivatives of delta function) injected at the detector. The parallel noise is by its origin a current source in parallel with the detector, and it is presented by a sequence of ***impulses*** (delta functions). *It is this difference in the location of the two white noise sources with respect to the detector capacitance that makes their apparent noise spectra and their effect on the measurement quite different.*

The series ***1/|f|*** noise contribution to ENC is independent of the peaking time and the time scale of the overall system response as long as the shape of the weighting function is preserved. This corresponds to a constant ratio of the high frequency and low frequency cutoffs, as indicated in Fig. 14. The ***1/|f|*** noise due to a dissipative dielectric depends on the dielectric loss factor ***tan(δ),*** as shown in Fig. 50. It can be significant with detector-amplifier connections on glass fiber circuit boards.

The expressions for the equivalent noise voltage ***spectral density*** at the input of a noiseless amplifier, and for the equivalent noise current spectral density as if originating in parallel with the detector capacitance are given in this figure. They are arrived at as follows:

1/f noise physical spectral density is defined as $A_f / f \; in \; [V^2/Hz]$.

"Physical" (unilateral) equivalent ***noise voltage spectral density*** due to all three noise sources defined above and illustrated in Fig. 6.11, referred to the input of the preamplifier:

$$v_n^2 = e_n^2 + \frac{A_f}{f} + 2q_e I_0 \frac{1}{C_{in}\omega^2}$$

"Physical" (unilateral) equivalent ***noise current spectral density*** into the input capacitance C_{in} obtained by differentiation of v_n^2, i.e., multiplying by $\omega^2 C_{in}^2$ in the frequency domain,

$$i_{neq}^2 = v_n^2 \omega^2 C_{in}^2 = \omega^2 C_{in}^2 e_n^2 + C_{in}^2 \omega^2 \frac{A_f}{f} + 2qI_0 = \omega^2 C_{in}^2 e_n^2 + 2\pi C_{in}^2 A_f \omega + 2qI_0$$

Fig. 17. The equivalent noise charge (ENC) is calculated by integrating the ***noise current spectral density multiplied by the square of the system response in either the time domain (weighting function) or the frequency domain (transfer function),***

$$ENC^2 = \frac{1}{2} e_n^2 C_{in}^2 I_1 + \pi C_{in}^2 A_f I_2 + qI_0 I_3$$

$$I_1 = \int_{-\infty}^{\infty} \left[w'(t) \right]^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left| H(j\omega) \right|^2 \omega^2 d\omega = \frac{A_1}{\tau}$$

$$I_2 = \int_{-\infty}^{\infty} \left[w^{(1/2)} \right]^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left| H(j\omega) \right|^2 \omega d\omega = A_2$$

$$I_3 = \int_{-\infty}^{\infty} \left[w(t) \right]^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left| H(j\omega) \right|^2 d\omega = A_3 \tau$$

where τ is the time width parameter of the weighting function, either the peaking time of the function, or some other characteristic time constant of the filter implemented in hardware.
I_1, I_2, I_3 are the noise integrals for the *series (voltage)* white noise and the series 1/f noise, and for the *parallel (current)* noise respectively. The integrals are derived in time domain from Campbell's theorem and expressed in frequency domain using Parseval's theorem [Appendix A1]. The coefficients A_1, A_2, A_3 are the ***form factors of the weighting function*** (and the equivalent transfer function). Their values for some frequently used functions are given in Fig. 22.

Fig. 18. Some more detail on the calculation of integrals I_1, I_3 and their approximate evaluation by piece-wise linear approximation of the weighting function:

Calculation of equivalent noise charge (ENC) for a signal processing chain described by a weighting function ***w(t)***. A bipolar weighting function is assumed in this case. The noise calculation is performed in time domain by using Campbell's theorem which was introduced in Fig. 11. For parallel noise, the superposition of effects of all random impulses leads to the expression for ENC_p^2, where $\overline{i_n^2} = 2\overline{n}q^2$, $\overline{n}$ = mean rate of impulses, q = impulse charge. For a current contributing full shot noise (all charge carriers

generated randomly) $\overline{i_n^2} = 2\, q_e I_o$, where q_e = electron charge and $I_o = \bar{n}\, q_e$ is the mean current. For a resistor in parallel with the detector $\overline{i_n^2} = 4\, kT/R_p$. $\overline{i_n^2}$ is the physical noise power spectral density. The weighting function is normalized to unity so that the definition of ***ENC is "the noise charge which produces an output of the same magnitude as an impulse signal of equal charge"***. For calculation of the series noise, we use the representation in terms of an equivalent current generator connected in parallel with the detector. A doublet $\boldsymbol{C_{in}\, \delta'(t)}$ acting upon the weighting function $\boldsymbol{w(t)}$ produces by convolution $\boldsymbol{C_{in} \cdot w'(t)}$. By integration of mean square contributions of all previous doublets, the expression for ENC_s^2 is obtained, where $\overline{e_n^2}$ is the noise spectral density for series noise in V^2/Hz. In terms of an equivalent series noise resistance $\overline{e_n^2}$ = 4 kTRs.

Noise contributions for both types of noise due to various segments (piece-wise linear approximation) of the weighting function are shown in the figure (expressions for integrals I_1 and I_3). In these calculations, either the impulse response of the system or the weighting function (its mirror image) can be used for time-invariant systems. ***For time-variant (gated or switched) systems, only a weighting function describes the performance correctly, while an apparent impulse response (waveform at the output) is not correct and can be misleading.*** Steepest parts of the weighting function contribute most to ENC_s, as they correspond to larger bandwidth. Flat parts do not contribute anything. In contrast, ENC_p is largely proportional to the width of the weighting function where it has any significant value.

Eqs. for ENC in Figs. 16-18 provide an insight into the general behavior of signal processing systems with respect to noise. The ENC due to series white amplifier noise is inversely proportional to the slope of the weighting function and therefore proportional to the bandwidth of the system. The ENC due to parallel white noise is proportional to the width of the weighting function and therefore to the overall integration time. If the weighting function form remains constant the ENC due to *1/f* noise is independent of the width of the weighting function, since the ratio of the high frequency cutoff and low frequency cutoff remains constant, Fig. 14. From Eqs. in Figs. 17 and 18 the optimum width parameter (known as the ***"noise corner time constant")*** of the weighting function is given by,

$$\tau_{opt} = \left(R_s R_p\right)^{1/2} C_{in} \left(A_1 / A_3 \right)^{1/2} ,$$

and it is not affected by *1/f* noise.

A bipolar weighting function, i.e., impulse response h(t), with equal lobes would result in square root of two higher ENC than for a unipolar function (single lobe). If the amplitude of the second lobe is less than one half, its rms noise contribution becomes small (<12%).

The half-order integral I_2 for *1/f* noise is not amenable to such a simple interpretation as for I_1 and I_3, and it will be discussed in Fig. 21.

Fig. 19. Following on the piecewise linear approximation of the weighting function in Fig. 18, a simple relation for the equivalent noise charge (ENC) due to series white noise is derived. It requires knowledge of three parameters: noise spectral density $\boldsymbol{e_n}$, total input capacitance (detector+amplifier) $\boldsymbol{C_{in}}$, and peaking time $\boldsymbol{t_m}$ of the triangle approximating the weighting function. Such an approximation is useful for noise estimation, since the series noise integral for a 5th order semi-gaussian weighting function with the same peaking time differs by only ~10%. In a preamplifier design, the expected $\boldsymbol{e_n}$ can be determined from the operating conditions (current and transconductance) of the first transistor.

Fig. 20. From Fig. 18 simple relations follow for ENC due to parallel shot noise and resistor (thermal) noise. The gated integrator case illustrates that the ENC for shot noise is simply the square root of the variance of a Poisson sequence of impulses counted for a time $\boldsymbol{t_G}$ (e.g., a gated integrator system). By Campbell's theorem the contribution of each impulse to the variance is determined by the weighting function, and for a given weighting function the parallel noise integral $\boldsymbol{I_2}$ has to be determined. For an approximation by a triangle with a peaking time $\boldsymbol{t_m}$, $I_2 = (2/3)\,t_m$. The parallel noise contribution for the triangular weighting function is the same as for gated integration one third as wide. The contribution by the parallel resistor thermal noise can be compared simply to the shot noise by the "***50 mv rule***": a dc current $\boldsymbol{I_0}$ causing a voltage difference of ~50 mV on a resistor $\boldsymbol{R_p}$ contributes equal amount of noise as the resistor at room temperature.

Fig. 21. Calculation and Estimation of ENC for 1/f Noise.

1/f noise becomes a limiting factor in many physical measurements. We can imagine reducing the series white noise in charge measurements to a very low level by continuing to increase the measurement (integration) time τ, provided the parallel (leakage or dark current) noise is very low. We would eventually reach the "noise floor" due to the *1/f* noise. Once the 1/f noise spectral density is determined experimentally and defined by the parameter A_f in [volts^2] as in Eqs. (5) and (6), ENC can be calculated by integral I_2, Eq. (8). In time domain this is an integral of a fractional-order (half-order) derivative squared of the weighting function (a mathematical operation which cannot be called "trivial" before one learns how to do it, and it can be considered "tedious" at best). In frequency domain the calculation is somewhat easier for time-invariant systems, but for time-variant systems defining the transfer function $H(j\omega)$ is more difficult and less intuitive than determining the weighting function.

We illustrate this here on the example of a commonly used weighting function of trapezoidal form . There are many different hardware implementations of this function in different applications. Time-invariant versions have used delay line clipping and higher order RC prefilters. Gated integrator and higher order prefilters have been used in several applications, starting with germanium gamma-ray detectors [Ref. 9]. This function is

widely used with CCDs in astronomy, implemented by correlated double sampling and dual-ramp integration.
We define the trapezoidal weighting function by the width of the ramp τ_p and the flat top as a fraction of the ramp, $\Delta\tau_p$. The equivalent noise charge for *1/f* noise is then,

$$ENC_f^2 = \pi C_{in}^2 A_f A_2 , \quad \text{where}$$
$$A_2 = \int_{-\infty}^{\infty} \left[w^{(1/2)}(t) \right]^2 dt$$

The coefficient A_2 vs the flat top Δ of the trapezoidal weighting function, according to an exact calculation (not reproduced here), is plotted in this figure.
The effect of the series *1/f* noise is lowest for a triangular weighting function, Δ =0. As the flat top is made longer, A_2 increases, since such a trapezoidal function has a higher ratio of its cutoff frequencies, which results in integrating a wider band of the *1/f* noise spectrum.

Since exact calculations of ENC_f for any weighting function can be time consuming, we emphasize here a simple estimation method, which provides results sufficiently close to the exact calculations for most purposes. It has been pointed out by Gatti *et al.* [42] that the three integrals in Fig.17 have to satisfy Cauchy-Schwartz inequality, which can be expressed in terms of the three weighting function coefficients (i.e., form factors),

$$A_2 \le \left(A_1 A_3 \right)^{(1/2)}$$

Thus there is an upper limit to A_2 in relation to A_1 and A_3 which are easily calculated from Figs 17 and 18, as illustrated by a simple case of a triangular function in Figs. 19 and 20. A study of the most commonly used weighting functions, reveals that $A_2 \big/ \sqrt{A_1 A_3}$ falls between 0.64 and 0.87, a spread of less than ±8% in the calculation of rms noise, so that for estimation of ***1/f*** noise the following approximate relation can be used,

$$A_2 \approx 0.75 \left(A_1 A_3 \right)^{1/2}$$

For the trapezoidal weighting function , $A_1 = 2$ and $A_3 = \left(\Delta + 2/3 \right)$, and the approximation for this case is,

$$A_2 \approx 0.75 \left[2 \left(\Delta + 2/3 \right) \right]^{1/2}$$

This approximation is plotted in the figure and it is within a few percent of the exact analytical solution.

In any noise analysis of charge amplifiers one will have already calculated, or otherwise determined the values of A_1 and A_3 , so that ***the information about the filtering (pulse shaping) effect on the series and parallel white noise will also provide an estimate of the 1/f noise***,

$$ENC_f^2 = \pi C_{in}^2 A_f A_2 \approx \pi C_{in}^2 A_f \left(0.75 \sqrt{A_1 A_3} \right)$$

It is interesting to note that calculated values are: for a gaussian weighting function A_2=1.00; for a triangular weighting function 0.88; for a 4th order semi-gaussian 1.02; for CR-RC 1.18.

A_f is a parameter resulting from a measured spectral density and it does not contain any specific information about the properties of the amplifying device unless other parameters are known.
For input transistor optimization a parameter which is to the first order independent of the device dimensions is more useful [15], $K_f = A_f C_{GS}$ [joule]. This constant ranges from 10^{-27}J for junction field-effect transistors (JFETs) to ~10^{-25}J for p-channel and ~10^{-24}J for n-channel MOS transistors.

The discussion here was intended to provide some insight: ***ENC$_f$ depends (rather weakly) only on the shape of the weighting function, but not on the time scale***.

Fig. 22. A summary of noise coefficients for some frequently used weighting functions.

Both calculated and approximate values for A_2 are given for comparison. ***ENC*** calculation is straightforward requiring the knowledge of the amplifier series noise spectral density e_n, total input capacitance C_{in}, shot noise contributing (leakage or dark) current I_0, and ***1/f*** noise spectral density parameter A_f.

Fig. 23. Noise corner frequency for ***1/f*** noise and white series noise for an NMOS transistor with a fairly high ***1/f*** noise. Transistor ***1/f*** noise is characterized by the constant K_F, which is independent of the device width and it is dependent on device technology [Refs. 15, 17]. This constant ranges from ~ 10^{-27} for the best JFETs to ~ 10^{-24} for some NMOS devices. The value for A_f is determined from the ***1/f*** spectral density (where the white noise is negligible) multiplied by the measurement frequency.

Fig. 24. Examples of the ***1/f*** noise corner measurement time for the equivalent noise charge (in terms of the weighting function peaking time) for different device technologies. The corner is off scale of this plot for JFETs. The well known difference between NMOS and PMOS transistors is apparent. In GaAs devices ***1/f*** noise is dominant to high frequencies, as $f_c t_{mc} = 1/4$. In BJTs series ***1/f*** noise is very small and not visible in this plot. The ENC for BJTs shows a different crossover – between the series (collector current) shot noise and the parallel base current noise, as indicated by the general plots In Figs. 17 and 18.

Fig. 25. Overview of white noise sources in an NMOS transistor normalized to the intrinsic channel series noise resistance γ/g_{ms}. The gate induced noise contribution with capacitive sources, such as is the case for most radiation detectors, is usually negligible. In particular, at operating conditions to minimize the power in the input transistor, the optimum ratio C_{gs}/C_{in} is small. The contributions by the gate resistance and substrate resistance can be made small by the device design.

4. Signal processing, i.e.,"filtering" or "pulse shaping"

Fig. 27. The concept of weighting function is very useful for time domain noise analysis of time variant, sampled and switched systems. The role of "pulse shaping," "signal filtering," or "signal processing" is to minimize the measurement error with respect to the noise, various baseline offsets and fluctuations, and at high counting rates to minimize the effects of pulse overlap or pileup. The term "pulse shaping" implies that the amplifier-filter system is time invariant. In such a system the system parameters do not vary during the measurement, and a single measurement of amplitude or time is performed. Such a system is described completely by its impulse response.
In signal filtering, we also use time-variant methods, such as capacitor switching and correlated multiple sampling of the signal. The filtering properties of a time-variant system are described by its weighting function ***w(t)***. ***The weighting function describes the contribution that a noise impulse, occurring at time t_i, makes at the measurement t_m***, as illustrated in the figure. It is essentially a measure of the memory of noise impulses (or any other signals) occurring before the observation time t_m. As shown, ***the weighting function for time-invariant systems is simply a mirror image in time of the impulse response, with its origin displaced to t_m***. For a time-variant system, the impulse response (output waveform) is generally quite different from its weighting function. Time-invariant and time-variant processing could be devised to produce the same result, i.e., both methods will be described by the same weighting function, while their implementation will be quite different. The noise-filtering properties of any weighting function for detector signal processing can be most easily determined by the time domain analysis technique shown in Fig. 19. Such a time domain analysis method was first introduced by Wilson [Ref. 3.], and subsequently elaborated on in [Refs. 5,6,17].

Fig. 28. In general, the composite weighting function for multiple correlated sampling is obtained by superposition of weighting functions for individual samples. This is illustrated for ***correlated double sampling (CDS)***, a technique commonly used for readout of CCD's and large pixel arrays. Single sample processing is described by a symmetrical triangular impulse response approximating single RC differentiation and one or two RC integrations. The single sample weighting function with respect to the sampling time at t_m is shown (dashed), and it is a mirror image of the impulse response. It is assumed that a (delta function) signal of interest will arrive at time t_{0+}, and produce a response described by the impulse response. In double correlated sampling another sample is taken at t_0, just before the arrival of the signal. This sample, sometimes called "baseline sample", is subtracted from the "signal or measurement sample". The weighting function for the baseline sample is shown inverted and earlier in time by t_m-t_0. The composite weighting function (thicker solid line) is bipolar and it has area balance. This is another way of saying that CDS has zero dc response and that it attenuates (but does not eliminate) baseline fluctuations at low frequencies. The ENC can be easily calculated from such a composite weighting function by using the technique for time

domain noise analysis shown in Fig. 19. (An analysis of such a case in frequency domain and without the use of composite weighting function is considerably more time consuming).

Fig. 29. A low order asymmetrical function (dashed) has higher series noise (steep rise, see Fig. 19). A nearly symmetrical function requires a higher order signal processing chain, as shown.

Fig. 30. An example of some commonly used functions for unipolar and bipolar pulse shaping (left) and their frequency response (right). The unipolar response is achieved with 5 RC integrations and 1 RC differentiation, with RC = 20 ns. The bipolar response is due to 4 RC integrations and 2 RC differentiations (equivalent to a mathematical differentiation of the unipolar response). The noise integrals from figure 17, $I_1 = A_1/\tau_F$ and $I_3 = A_3\tau_F$, form factors A_1 and A_3, and τ_F = FWHM (approximately equal to zero-to-peak time) are:

	τ_F	I_1 [s^{-1}]	I_3 [ns]	A_1	A_3
unipolar	105	2.2×10^7	80	2.3	0.76
bipolar	52	6.2×10^7	58	3.2	1.1

Vertical scale for frequency response is in decibels (20 db = factor of 10 in amplitude). The series noise ENC_s^2 is higher for bipolar function of the same width as the unipolar one. As this function has area balance the response at low frequencies is significantly lower, and so is the sensitivity to low frequency baseline fluctuations, pileup and noise.

Fig. 31. Impulse response *h(t)* of (a) the optimum filter for amplifier (series) white noise and (parallel) white noise current at the detector electrode. The filter requires a long time to respond. (b) Triangular impulse response optimized in width with respect to the noise corner time constant, given by $\tau_c = C_{in}\left(R_s R_p\right)^{1/2}$. The noise is higher by only 8% compared with the ideal filter. The width of both responses is determined by the noise corner time constant τ_c. The time of the amplitude measurement (peak of the response) is at the origin. (c) Trapezoidal weighting function provides uniform weighting at the flat top to reduce ballistic deficit for signals of finite width. Implemented with a gated integrator [Ref. 9.].

Fig. 32. Schematic representation of basic digital functions in sampled data signal processing. An anti-aliasing filter is an essential part of the system. Its function is to limit the bandwidth prior to sampling so as to satisfy the Nyquist sampling criterion: the bandwidth at the output of this filter must be no more than one half of the sampling frequency. If this is not satisfied, the loss in S/N due to aliasing cannot be recovered by any subsequent processing. The role of digital filtering in x-ray and gamma-ray

spectrometry is to create optimized weighting functions. In spite of the power of digital processing it is most efficient to cancel any long tails in the detector-preamplifier response by analog means. If the tail cancellation is performed digitally, much larger numbers of samples have to be processed (deconvolved) for each event. For asynchronous (uncorrelated) sampling for semiconductor detectors see [Ref. 19]. For signal processing with multiple synchronous (correlated) samples for liquid argon calorimeters see [Ref. 22].

Fig 33. An example of reconstituted waveforms illustrating formation of trapezoidal functions, which can be adapted in their width to the counting rate. The top trace is the digitized preamplifier output with numbered x-ray steps. The middle and bottom traces are the corresponding fast and slow filter outputs. X-ray 1 is adequately isolated, so the slow filter output (with better S/N) is not affected by the subsequent event 2. Pulses 2 and 3 are resolved by the fast filter but pileup in the slow filter. Pulses 4 and 5 pileup in the fast filter, and are rejected on the basis of the fast filter pulse width. The sampling period in asynchronous (uncorrelated) sampling has to be much shorter than the peaking time of the required weighting function. For example, in high resolution gamma-ray spectrometry, 50 ns sampling period may have be used for peaking times in the microsecond range.

Fig. 34. There are various techniques to try to "restore", "correct" or "stabilize" the baseline when measuring random signals from detectors. These techniques often involve time-variant or nonlinear operations, and an exact characterization of their properties is not straightforward. Baseline restorers perform a part of the function of correlated double (or even triple) sampling – taking and storing the first sample before (or just at) the beginning of the signal response, i.e., the baseline sample, at t_2 in Fig. 34.1. The signal with the baseline is then measured at t_3 . This 1st order baseline restoration corrects for dc offsets, and for slow variations in the baseline (due to previous signals, hum, etc.) relative to the time between the two samples. It is usually implemented by a nonlinear active or passive diode circuit as in (a) where the baseline sample is stored on C as soon as the signal starts rising. (b) and (c) illustrate gated restoration, where the gate is derived from a separate timing signal. In all three cases the signal appears at the output with the initial value of the baseline subtracted. For a higher order of low frequency rejection triple correlated sampling would provide for baseline slope correction. A second order restorer was developed in one case for this purpose, [Ref. 39]. Fig. 34.2 shows a feedback circuit for "stabilization" of the baseline. If such a circuit is linear, it represents "ac coupling in disguise", and no other sampling or restoring function. If nonlinear, restorer functions can be configured, as in [Ref. 9].
The noise performance of all such circuits can best be characterized by determining their weighting function. An example how to do this is given in [Ref. 39]. The weighting function for circuits where nonlinear components play a role, will be different for small and large signals. The weighting function for small signals is relevant for the noise analysis.

5. Charge Amplifier Configuration

Fig. 36. In the most basic charge amplifier feedback configuration only two transistors are essential to realize a complementary cascode. The current sources in positive and negative supplies can be realized by resistors or by low noise transistor current sources. There is only one significant pole (C_oR_o) in this circuit. Higher order poles are given by the unity gain frequency of the transistors used. The cascode alone is an "operational transconductance amplifier" (very high output impedance). With the follower amplifier x1 it becomes an operational amplifier.

<u>Gain and input impedance relations for the feedback charge amplifier configuration .</u> The frequency dependence of the open loop gain is inherent to a high gain single pole amplifier. It is described by two parameters, unity gain frequency $\omega_h = g_m/C_o$, and the gain "roll off" frequency (3 db point) $\omega_l = 1/R_oC_o$. The dc gain is then $|G_o| = \omega_h/\omega_l = g_m \cdot R_o$. g_m is the transconductance of the input transistor, C_o is the dominant pole capacitance and R_o is the dominant pole resistance. Input impedance with capacitive feedback has two terms, a resistance $R_{in} = 1/\omega_hC_f$ in series with a capacitance C_fG_o. The resistance term R_{in}, in conjunction with the total input capacitance, determines the rise time of the detector-amplifier. The rise time constant of the output voltage (i.e., the transfer of charge from the detector capacitance to the feedback capacitance) is $\tau_r = R_{in} \cdot C_{in} = (1/\omega_h)\ (C_{in}/C_f) = (C_o/g_m)\ (C_{in}/C_f)$, where $C_{in} = C_{Det} + C_{ampl}$. The resistive input impedance has a noise corresponding to the amplifier series noise resistance R_{seq}, and it appears as a resistance with a noise temperature, $T_{eff} = T \cdot R_{seq}/R_{in}$. For values of R_{in} higher than R_{seq}, the amplifier can be used as a termination for delay lines with a noise lower than that of a termination with a physical resistor Z_o at temperature T. The noise temperature of the resistance Z_o realized by the capacitance in feedback is $Z_o \cdot R_{seq}/Z_o$, and this is why it can be called ***"electronically cooled termination***" or ***"electronically cooled damping",*** [Ref. 11]. The resistance in parallel with the feedback capacitance adds two more terms to the input impedance of the preamplifier: inductance R_f/ω_h in series with a resistance R_f/G_0. It is important to note the condition to achieve an aperiodic ("damped") response of the feedback amplifier.

The feedback configuration allows the ultimate in noise performance because the parallel noise sources can be made negligible by using a transistor with a very low gate leakage current and a very high feedback resistance (megaohms to gigaohms). The feedback resistor can be avoided altogether by the use of optoelectronic feedback or a transistor switch to maintain amplifier voltages in the operating range. Signal integration is performed on the feedback capacitance C_f. The long tail can be cancelled in subsequent pulse shaping by a simple pole-zero cancellation circuit (not shown in the figure). Pulse shaping at the preamplifier by reducing R_L or R_f would result in increased noise from the thermal noise of these resistors. The object of the design is to avoid dissipative components at the detector-amplifier input and thus to make R_f as large as possible.

Fig. 37. Transfer of charge from detector to amplifier.

In all cases where the amplifier is connected directly to the detector via a resistive conductor the charge produced by ionization is distributed among the detector capacitance, amplifier capacitance and any stray capacitance according to the ratio of capacitances. Due to this, only a fraction of the charge of interest (the signal) arrives where it matters – that is to the conduction channel of the input transistor where it controls the drain (collector) current. (An exception to this is if the two capacitors are connected by an inductor in which case the charge is transferred periodically between the two capacitors.) In case of a CCD the ionization charge is moved peristaltically in a potential well formed and driven by appropriate clock voltages applied to the gate electrodes. The charge shifted a few hundred (or thousand) times arrives at the collection electrode ("floating diode") which is connected to a source follower. In the CCD the charge arriving at the collection electrode is the original charge packet produced by ionization except for a few electrons lost to trapping. The charge transport in a conductor is by a small displacement of a large number of free electrons. CCD principle allows multiple measurements on the same charge packet as explained by Fig. 38.
Optimization of the signal to noise ratio requires appropriate matching of the transistor active capacitance (which controls the current) to all other capacitances connected to the input – a subject addressed in the section on ASIC design.

Fig. 38. Charge coupled devices (CCDs) make possible by their principle an additional degree of freedom in the processing of their signals. There is a fundamental difference in the treatment of charge produced by ionization (or photo emission) between the devices based on ***charge transfer*** and all the detectors based on ***collection of charge***.
Collection of charge allows only "one look" at the charge, no matter for how long (weighting function width to optimize the noise). Charge transfer allows a charge packet to be moved back and forth in the proximity of the sense electrode, making possible "multiple looks" at the induced charge, i.e., a repetitive measurement. If we impose the same total measurement time on both types of measurements (single long time vs repetitive short times) the result for ENC due to the series white noise will be the same. However, the ENC for ***1/f*** noise will be reduced in the repetitive measurement. We know that due to the scaling invariance of ***1/f*** noise, a single measurement is independent of the shaping (integration) time, as the filter passband moves up or down the ***1/f*** spectrum, while the ratio of the upper and lower cutoff frequency remains constant. In contrast, in the repetitive measurement, averaging of samples taken at a frequency f_0 performs narrowing of the bandpass around the repetition frequency. The higher the repetition frequency, the lower is the ***1/f*** spectral density for each sample. An example of ENC improvement by this technique is shown in Fig. 34. This method made possible the lowest read noise achieved in a CCD readout of less than ***1 e rms***, [Ref. 40].

6. Timing Measurements

Fig. 40. The timing error ("timing jitter") due to electronic noise in any timing measurement is given by the ratio of the amplitude noise (ENC) and the slope of the signal at the crossing of a discriminator threshold. This figure illustrates qualitatively the timing "walk" problem, that is the dependence of the threshold crossing time on the signal amplitude. For signals with negligible waveform fluctuations, amplitude dependence in timing can be eliminated by bipolar pulse shaping and the timing signal derived from the zero crossing, which has a well known antiwalk property. Alternatively, when both amplitude and time measurements are performed the timing walk can be corrected in subsequent processing.

Derivation of an optimal weighting function which minimizes the timing error with respect to noise for ***a known and constant signal waveform*** follows from the derivation of optimum filtering in amplitude measurements. In general, ***the optimum filter (weighting function) for timing is the derivative of the optimum filter for amplitude measurements***. From a unipolar filter for amplitude measurements this results in a bipolar one with zero crossing, which has antiwalk properties.

Fig. 41. Constant fraction timing was developed to address two questions: 1) timing of signals with waveforms varying due to either the dependence of the position of the origin of the charge in the detector, or the statistical fluctuations; and 2) deriving of the timing mark early at the beginning of signals of extended duration. The method that solves 2), is also a solution for 1).

A ***constant fraction (CF) timing signal*** is formed by combining the ***delayed signal*** with the ***attenuated inverted (undelayed) signal***.

The first column in this figure illustrates this for a detector current of indefinite duration. The second column is for typical *integrated currents* from germanium detectors. In both cases antiwalk properties of the CF signal are apparent, and in the second column, shape invariance for germanium detector signals is also illustrated. In the third column CF signals are shown for a signal at the output of a typical pulse shaping circuit.

CF signals have a zero crossing from which the timing mark is derived, but for CF<1 they do not necessarily have area balance (CF=1 corresponds to simple delay line clipping which provides area balance). For CF<1, area balance can be achieved by an additional RC differentiation.

The smaller the CF, the earlier is the timing mark derived. As the CF signal before the zero crossing has only a fraction of the original signal amplitude, the lower limit to CF is set by noise (frequently one would like to time on the first photo electron - but for the noise).

This method for timing on the very early part of the signal requires the use of a non-dispersive delay (i.e., an electromagnetic delay line).

Fig. 42. It is possible to form a constant fraction (antiwalk) signal by RC networks (the only way realizable in monolithic circuits) , but a penalty is that the timing mark is obtained late (after the peak of the original shaped signal). The signal cannot be delayed by RC networks, only its shape can be changed to allow formation of a CF signal. Nevertheless, the timing resolution can be very good with this method, [Refs. 23,24].

7. Fluctuations on a Capacitance and Charge Measurement Sensitivity – kTC Noise

Fig. 44. Integration of the power spectrum (spectral density) arising on a capacitance from the thermal noise of a resistor results in the total fluctuation of charge (and voltage), which is independent of the value of the resistance ***R***. The resistance (with the capacitance ***C***) determines the bandwidth of the noise. The ***kTC*** charge noise is quite high even on small capacitances. Most of this noise does not affect the measurement in systems where filtering and a very high parallel (feedback) resistance can be used. Such a system "looks" at the portion of the spectrum where the spectral density is very low. An example: high resolution x-ray spectrometry with silicon detectors. When the measurement is performed directly on the detector capacitance and the filtering is not possible, the full ***kTC*** noise is included in the measurement, and it can be reduced only by correlated double sampling - if applicable, as discussed in *Fig. 48*.

The equipartion theorem of statistical mechanics, assigns a fluctuation energy of $(1/2)k_BT$ to each degree of freedom of a system in thermodynamic equilibrium, equal to $(1/2)(\sigma_q^2/C)$ on a capacitance, giving the same result as the RC circuit calculation (integration of the entire noise spectrum). The equipartition theorem does not imply that there is a resistance in the circuit.

Fig. 45. Charge and Voltage Total Fluctuations on Capacitance at 300K

Fig. 46. Transient behavior of noise after circuit parameters have been changed by switching can best be studied by applying Campbell's theorem, as shown in the figure. The oscillogram shows build up of noise after switching a white noise source onto an RC circuit. Such a build up occurs after a reset switch across a capacitor is opened. This is illustrated some more in *Figs. 47 and 48*.

Fig. 47. Charge transfer and build-up of ***kTC*** noise in active pixel sensors and the readout node of CCDs is shown after the reset transistor ***(***or charge transfer transistor) is turned on. By the time most of the previously stored charge is reset (or transferred) the noise is already fully built up. Resistance ***R*** in the expressions for σ_q and $q_s(t)$ is the ON-resistance of the transistor switch. The noise in this case is often referred to as the "reset noise", even though its origin is not in the reset action.

Fig. 48. This illustrates more completely what happens with ***kTC*** noise in active pixel sensors and CCDs. While the reset switch is closed the ***kTC*** noise extends to very high frequencies corresponding to the very short time constant $r_{ON}C$. When the switch is "opened" the time constant increases by many orders of magnitude. A value of the "old" ***kTC*** noise is stored on the capacitance, and it decays very slowly with this very long time constant. At the same time the "new" ***kTC*** noise builds up also very slowly, but faster than the stored value decays, as shown in *Fig.47*. From this one can see the conditions under which correlated double sampling may reduce significantly the ***kTC*** noise: ***Sample 1 may be taken any time between opening of the reset switch and the arrival of the signal. Sample 2 may be taken any time after the arrival of the signal but before the "new" kTC noise has built up.***

Some general remarks on ***kTC*** noise:

1. *Fluctuation-dissipation theorem* with Nyquist's expression for thermal noise is essential for calculation of noise spectra and for detailed information on noise sources.
2. *Equipartition theorem* provides no detailed information, but provides a *check on the integrals* of noise spectra (the total fluctuation).
3. *Transient behavior* of noise in switched capacitor circuits and matrix readout pixel arrays is best understood by means of *Campbell's theorem*, which provides *noise variance vs time*, as shown in *Figs. 46, 47*.
4. A complete charge reset and charge transfer by a switch result in $\sigma_q^2 = k_B TC$, independently of the switch ON resistance. This noise can be subtracted only if the *first sample* in the CDS is taken *before the signal*.
5. Transfer (i.e., direct transport) of charge without switching (as in a CCD) does not result in $k_B TC$ noise. *Reset* of the sense amplifier *does*.

8. Noise from Dielectrics

Fig. 50. Thermal fluctuations in dielectrics generate electric noise the magnitude of which is related to the loss conductance of the dielectric by the dissipation-fluctuation theorem. This noise was first measured in charge amplifiers and analyzed in [Refs. 7 and 8]. For a dielectric with low losses, the dissipation factor is a constant over the frequencies of interest,

$$D = G(\omega)/\omega C_d,$$

where $G(\omega)$ and C_d are the loss conductance and the capacitance of the dielectric as measured on an impedance bridge at an angular frequency ω. The conductance $G(\omega)$ generates a noise current at the frequency ω, according to Nyquist's formula,

$$\overline{i^2(\omega)} = 4kT\, G(\omega)df = 4kT\, (D\, C_d)\omega\, df$$

This current is converted into a noise voltage on the input capacitance,

$$\overline{v^2(\omega)} = (4kT\, D\, C_d/\omega)\ (df/C_{in}^2)$$

The lossy dielectric generates an $|f|$ noise current spectrum in parallel with the input, which integrated on the input capacitance becomes $1/|f|$ noise. The equivalent noise charge due to this noise can be calculated by integrating the noise voltage at the output of the filter with respect to frequency and comparing it to the signal output. One obtains for the equivalent noise charge,

$$(ENC)^2 \approx 2.4\ kT\ (D\ C_d)\ .$$

An interesting characteristic of the dielectric noise is that its contribution is independent of both the input capacitance (except for the capacitance of the dielectric) and the filter time constant. It is also little dependent on the type of the filter (weighting function). In cases where the dielectric and/or its capacitance are not known, it is more convenient to use the loss conductance as measured on an impedance bridge at an arbitrary frequency f_o in the range of interest. Then,

$$DC_d = G(f_o)/2\pi f_o \quad .$$

Such noise arises in conventional transistor and IC plastic packages with relatively high losses. The noise contributions of these packages at 295°K have been found to range from about 25 rms electron charges for the very best to more than 100 rms e for the worst. A reduction by a factor of 2 to 3 in the rms equivalent noise charge was observed at 125°K. Glass fiber (G10, Fr4) circuit boards have relatively high dielectric losses (D~2%).

A stray capacitance of ***1pF*** of a trace on such a board at ***293 K*** contributes $\boldsymbol{ENC_D \sim 80\text{-}90}$ ***rms e!***

Clearly, detector-amplifier connections should be on low loss dielectrics (e.g., Teflon board), and the stray capacitance should be minimized.
Insulating materials even in the vicinity of the input lead should be kept to a minimum, as their noise is coupled capacitively to the input.

Calculation of the total fluctuation for complex networks (electrical and mechanical) shows that it (the integral of the noise spetrum) is given by the equipartition theorem (*Figs. 44, 45*).

9. Signal Return in Multi-Element Detectors

Fig. 52. In large multi-wire chambers with 2-dimensional readout the signal return path is essentially from the readout of one coordinate to another and it could be very long. Amplifier "commons" have to be connected to each other by a low impedance path. Signal reflections on long wires may be significant and impedance terminations would contribute to the overall noise. The effect of the reflections becomes negligible if the pulse shaping (peaking time) satisfies the condition, $t_p \geq 4t_D$, where $\boldsymbol{t_D}$ is the propagation time in the longest electrode.

In finely segmented (pixel) detectors (gas, liquid and semiconductor) crosstalk is introduced if the signal is not returned by the opposite electrode via a low impedance path. Opposite electrode is frequently left floating due to the difficulties in providing a bypass with a sufficiently large capacitance (and a low inductance!). Such a crosstalk is small for single particles and for a large number of pixels, but it is significant at high particle multiplicities or in high resolution x-ray detectors. To minimize the energy stored on a large bypass capacitance, an active bypass may be used, as in *Fig. 39*.

A related subject, grounding and shielding in large detectors is discussed in [Ref. 28].

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Appendix

A.1. Noise Calculation : Time Domain and Frequency Domain,

Fig. 58.

A.2. Parallel Noise due to Dark Current in Devices with Avalanche Gain

Fig. 59. *Important to note*: If we measure a dark current I_0 from an avalanche photo diode, then the shot noise spectral density is not $\overline{i^2} = 2eI_0$ **,** but rather, $\overline{i^2} = 2eI_0 \cdot M$. Consequently, the ENC^2 is magnified by the avalanche gain M .

A.3. Noise Figure vs Noise Voltage, Noise Current and Noise Temperature

Fig. 60. Characterization of amplifier noise in terms of the noise figure applies to resistive signal sources and not to capacitive ones.

A.4. Noise in Resistors at Different Temperatures and "Electronic Cooling" by Feedback

Fig. 61. The total fluctuation (by equipartition theorem) is a function of the temperature of dissipative components, and the effective temperature for a combination of resistors at different temperatures is easily determined. We have seen in *Fig.36* that a resistance can be realized by capacitive feedback around a transconductance amplifier (the basic charge amplifier configuration), $R_2 = R_{in} = (1/g_m)(C_0/C_f)$. The noise associated with this apparent resistance arises from the amplifier equivalent series noise resistance. It can be shown that the effective noise temperature $\boldsymbol{T_2}$ of $\boldsymbol{R_2}$ is given by [*Ref. 11*],

$$T_2 = (R_{na}/R_2)T_1 \ ,$$

where $\boldsymbol{R_{na}}$ is the amplifier equivalent series noise resistance at temperature $\boldsymbol{T_1}$.
The total fluctuation is then reduced by a factor determined by the value of the resistance to be achieved by feedback in relation to the amplifier series noise resistance.
Such a resistance can be used as "electronically cooled" damping or delay line termination with a lower noise than with a physical resistor. Since the "cooling" is achieved by using an active device, such a system is not considered to be in thermal equilibrium.

A.5. Zero-Crossing Statistics of Noise

Fig. 62. Knowledge of noise zero-crossing rate is needed in self-triggered systems when the detection threshold has to lowered to increase the detection sensitivity until the noise counts exceed a desired limit. As shown, the zero-crossing rate is defined by the second moment of the noise frequency spectrum. Due to this the zero crossing rate is affected by the shape of the spectrum as it approaches its high frequency cutoff. For an $(RC)^2$ low pass filter applied to white noise, the zero crossing rate equals the 3db cutoff frequency. For a uniform spectrum up until an abrupt (or high order) cutoff at $\boldsymbol{f_h}$, the zero crossing rate is $n_{zc+} = f_h/\sqrt{3}$. A discriminator level crossing rate ("noise counts") is determined from the gaussian distribution, as shown in the figure. At ~5 times the rms noise, the level crossing rate is reduced by more than 5 orders of magnitude from the zero-crossing rate.

A.6 Noise Autocorrelation Function as a Diagnostic Tool

Fig. 63. Calculation of correlation functions for more sophisticated filters than the CR-RC filter becomes tedious and the expressions are rather involved. In such cases, as well as in cases where the noise spectrum is not exactly known, measurement of correlation functions might be simpler or necessary. A method is described here which requires no additional equipment except an oscilloscope.

It is known from random noise theory that the ensemble average value $\overline{X}(r)$ of a large number of samples, each of them taken at time $\boldsymbol{\tau}$ after the noise waveform has crossed some fixed level $\boldsymbol{X_o}$, is given by the correlation function,

$$\overline{X}(r)/X_o = k(\tau) = K(\tau)/K(0) \ .$$

The correlation function can, in turn, be interpreted as the mean of the conditional probability density function that the noise will have value $\boldsymbol{X = X_o k(\tau)}$ *at time* $\boldsymbol{t = \tau}$, if it had value $\boldsymbol{X_o}$ at time $\boldsymbol{t = 0}$.

It follows from the above that if a trigger signal is derived for the time base of an oscilloscope, each time the noise crosses some predetermined level X_o - both with positive and negative slope - the mean value of the recorded oscillogram would represent the correlation function of noise. The variance of that mean value is given by

$$\sigma_k^2 = K(0) - K^2(\tau)/K(0) = \sigma^2[1-k^2(r)] \ .$$

As the correlation function $K(\tau)$ approaches zero the variance of the mean approaches the variance of the noise. Therefore, to make the amplitude of the observed correlation function high compared to σ_k, trigger level X_o should be set high. However, the number of crossings of that level decreases as a Gaussian function of $X_o/\sqrt{2}\sigma$, where σ is the rms value of noise, see *A.4* and *Fig. 64*. It was found that for $X_o/\sigma \sim 4$ *the number of* X_o crossings is large enough for observation and recording of noise in a short time.

Fig. 64. An Example of Noise Autocorrelation Measurement by Digital Oscilloscope

While with an analogue oscilloscope the mean value of the (auto) correlation function has to be estimated by eye, as in [*Ref. 41, which also gives further references*], modern digital oscilloscopes perform averaging of noise waveforms, and a function with negligible fluctuations is obtained. This figure shows an example of atypical autocorrelation function for white series noise after a semigaussian filter.

A.7. Pileup Effect on Centroid Position Resolution for unipolar and Bipolar Shaping (Weighting Function)

Fig. 65. Experimental results show a significant reduction in pileup effects with bipolar (area balanced) weighting functions.

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