DETECTOR SIGNAL PROCESSING

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1.Signal Formation and Ramo's Theorem

2. Noise Generation Mechanisms

- Shot noise and Campbell's theorem
- Generation of noise spectra
- Random telegraph Noise (RTS) and 1/f noise

3.Equivalent Noise Charge (ENC) Calculation

- ENC calculation in time and frequency domains
- Simple ENC calculation for series and parallel noise
- ENC Calculation for 1/f noise
- Noise corner frequency and noise corner time constant for 1/f vs. white noise

4.Signal Processing, i.e., "filtering" or "pulse shaping“

- Weighting function of the whole detector readout system
- Time invariant pulse shaping
- Correlated and uncorrelated sampling
- Optimum weighting functions
- Correlated sampling by baseline restoration

5. Charge Amplifier Configuration
Transfer of charge from detector to amplifier
6. Timing Measurements

Anti-walk techniques

7. Total Fluctuations on a Capacitance and Charge Measurement Sensitivity

"kTC noise" and correlated sampling

8. Noise from Dielectrics

9. Signal Return in Detectors

10. References on Signal Processing for Radiation Detectors

Appendix

A.1 Noise calculation: Time domain and Frequency domain
A.2 Parallel Noise in devices with Avalanche Gain
A.3 Noise Figure and Noise Temperature
A.4 Noise in Resistors at Different Temperatures and "Electronic Cooling" by Feedback
A.5 Zero-crossing statistics of noise
A.6 Autocorrelation function as a diagnostic tool
A.7 Pileup effects

1. Signal Formation and Ramo's Theorem

Induced Current and Charge

S. Ramo, Proc. IRE, 27(1939) 584
(and W. Shockley)

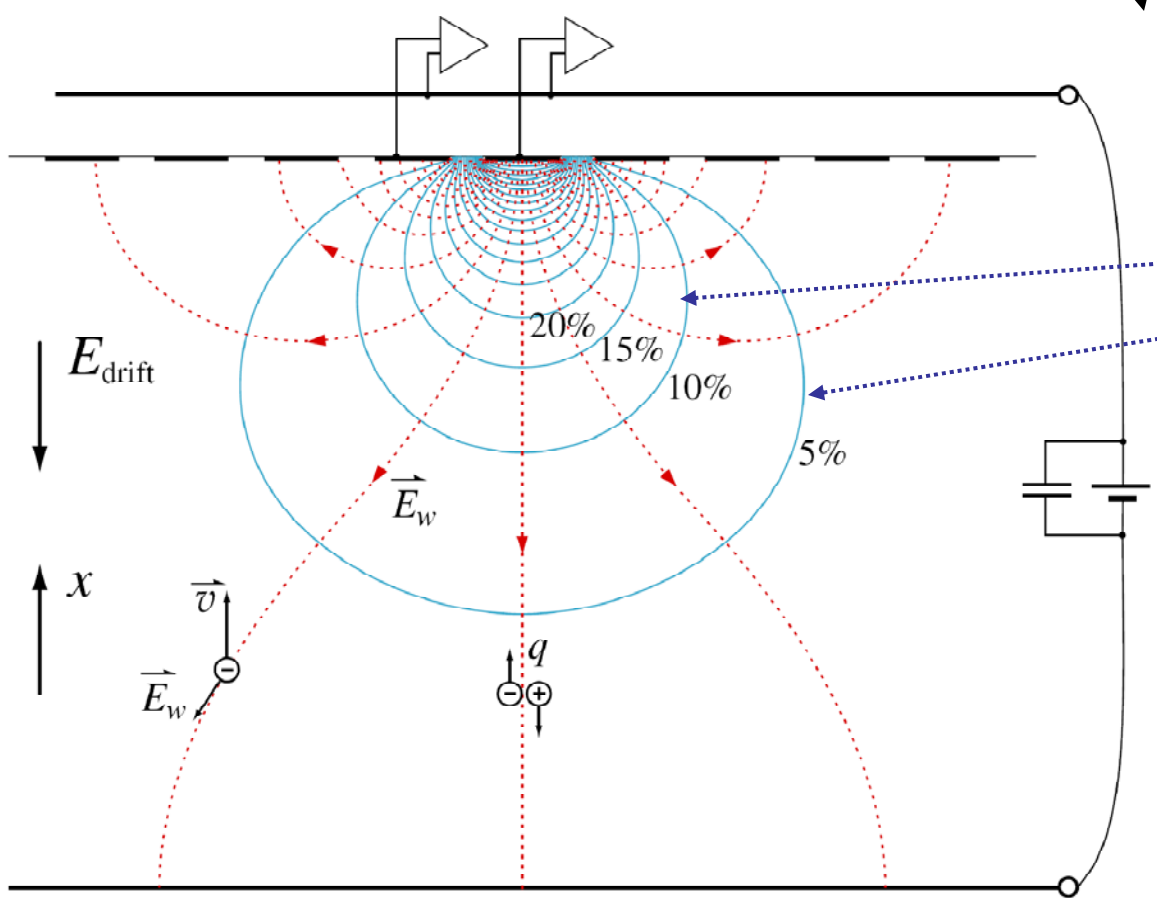
Weighting field

$$i = -q\vec{E}_w \times \vec{v}$$

$$v = dx/dt$$

$$Q_s = \int i dt = +q\int \vec{E}_w d\vec{x}$$

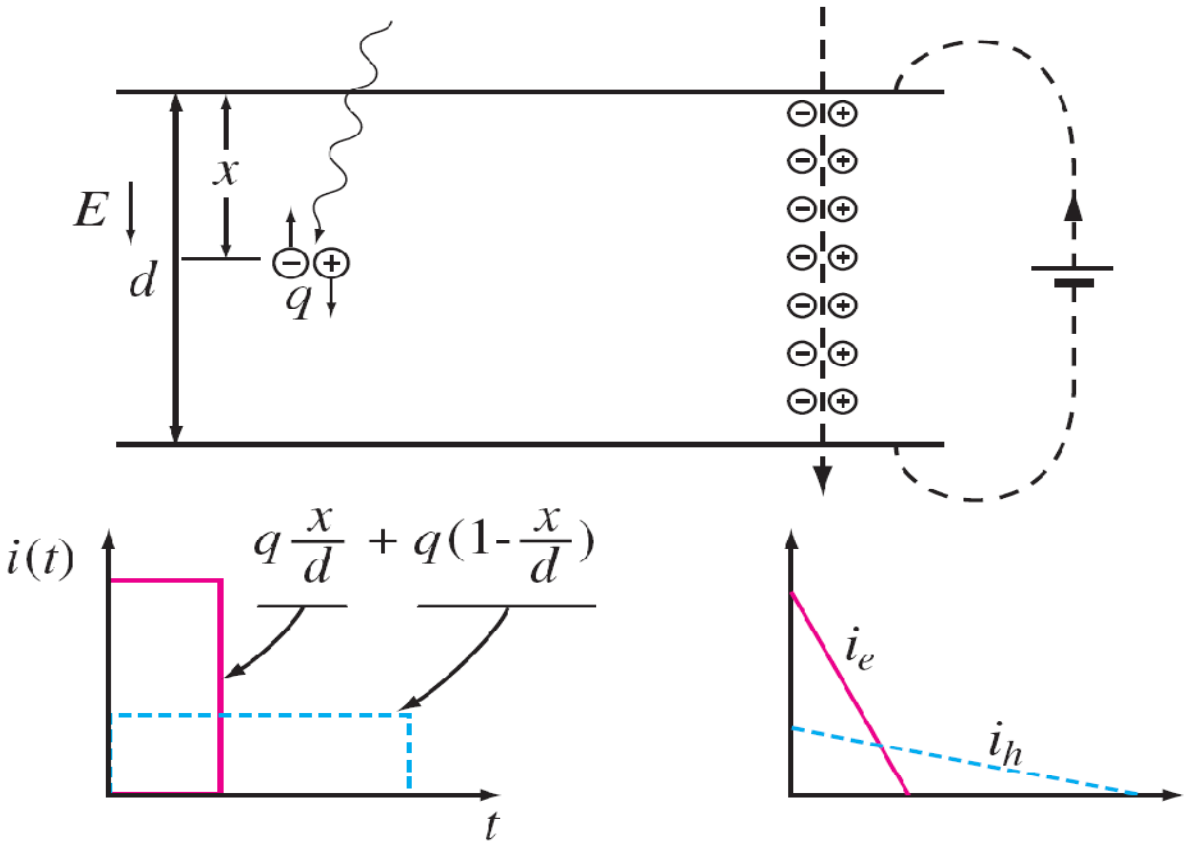
$$Q_s(1,2) = q(V_{w2} - V_{w1})$$



Weighting potentials

4

The Simplest Case: Induced Currents in Continuous Planar Electrodes for Localized and Extended Ionization in Semiconductor, Gas and Liquid Detectors



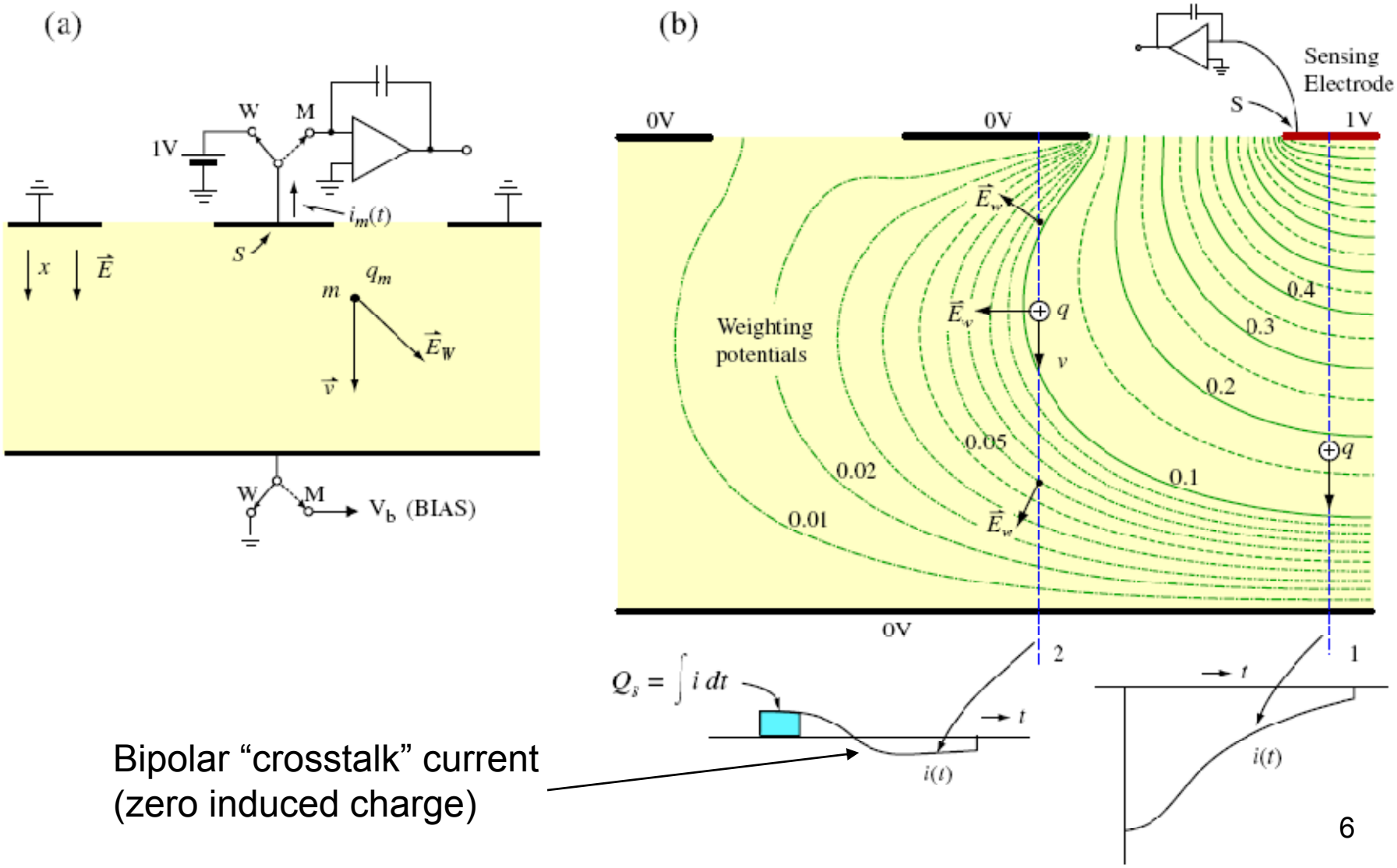
$$E_w = \frac{1}{d}$$

Carrier drift velocity

$$i = q \cdot \frac{v}{d} = \frac{q}{t_d}$$

Carrier transit time

Induced Currents in Strip Electrodes



Bipolar "crosstalk" current
(zero induced charge)

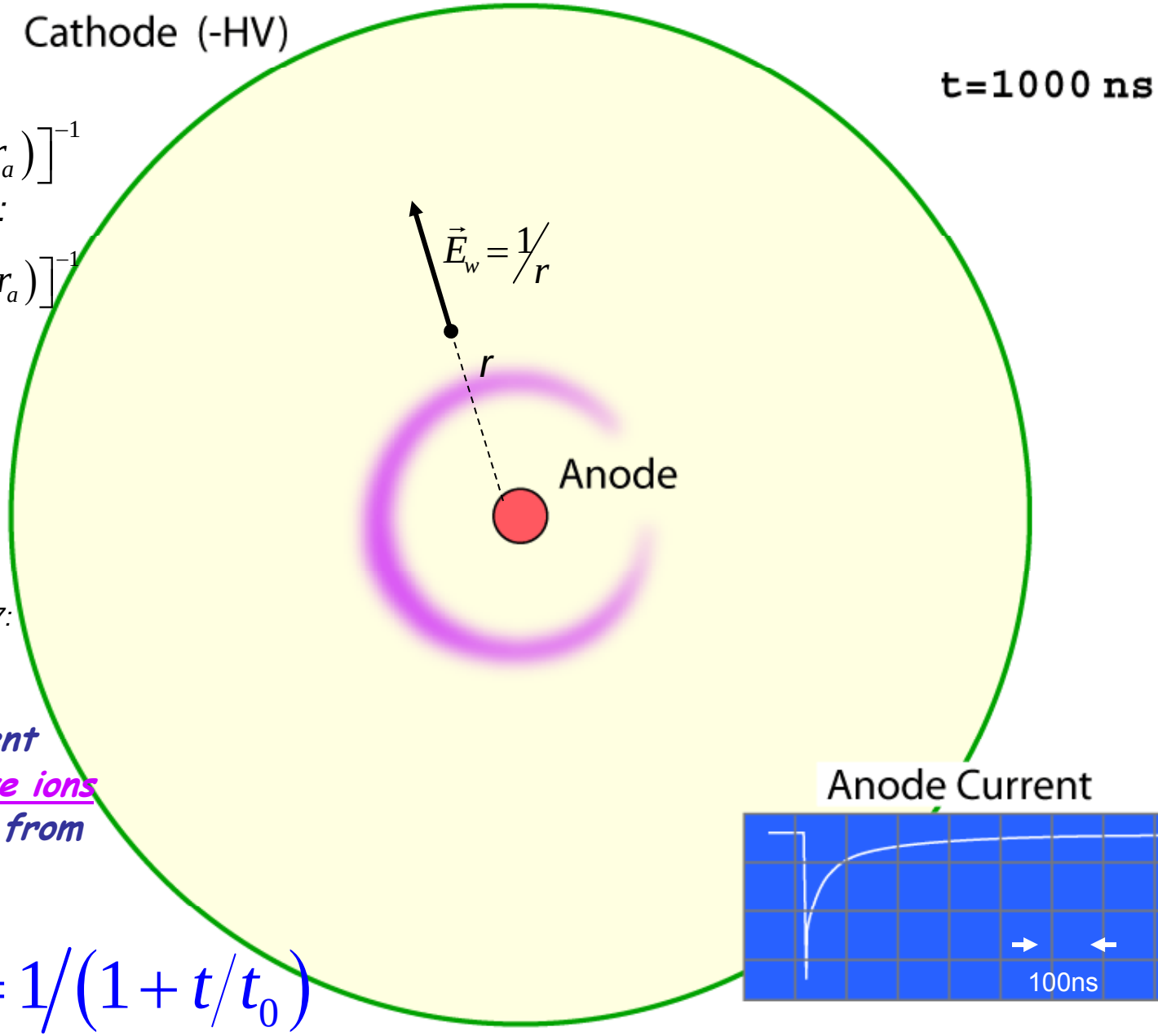
Operating Principle of a Proportional Counter

Weighting field:

$$E_w = \left[r \ln\left(r_c / r_a\right) \right]^{-1}$$

Operating field :

$$E = V \left[r \ln\left(r_c / r_a\right) \right]^{-1}$$

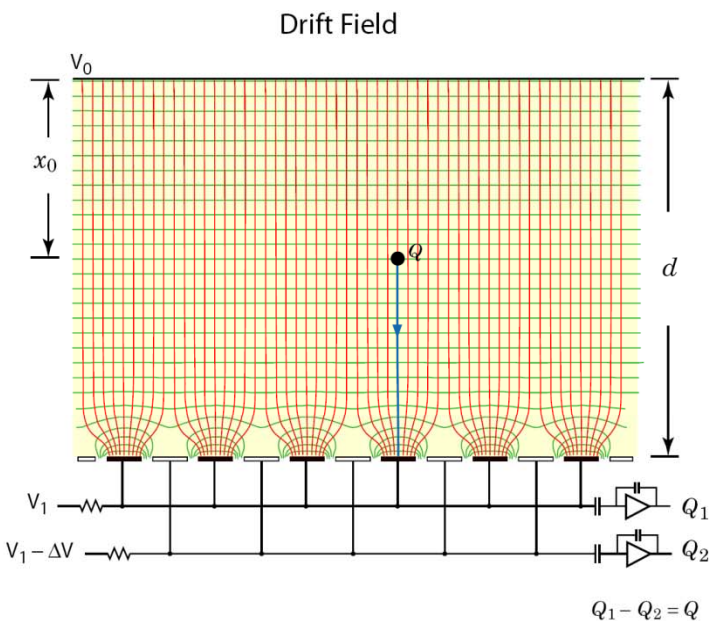


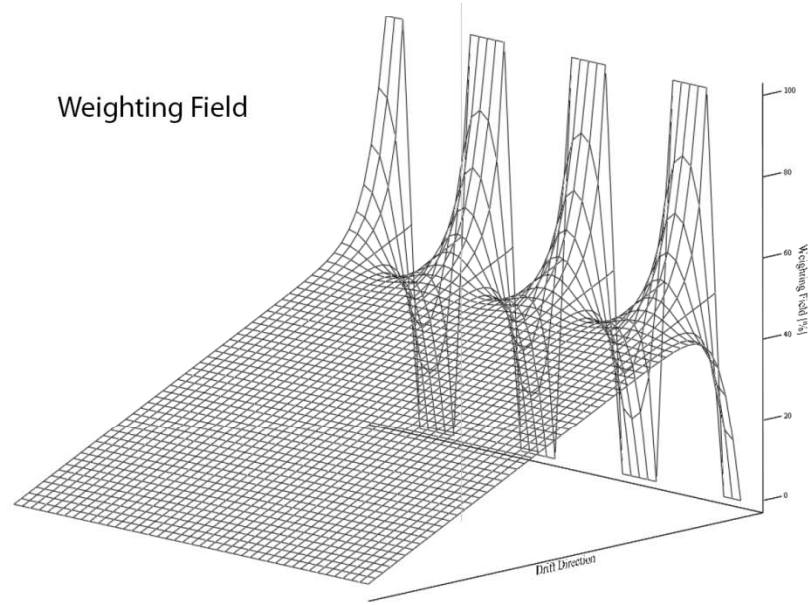
Derivation in: Ref. 17:

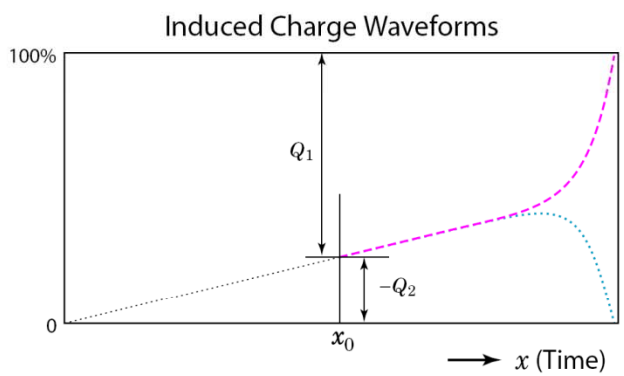
Induced current due to positive ions drifting away from the anode:

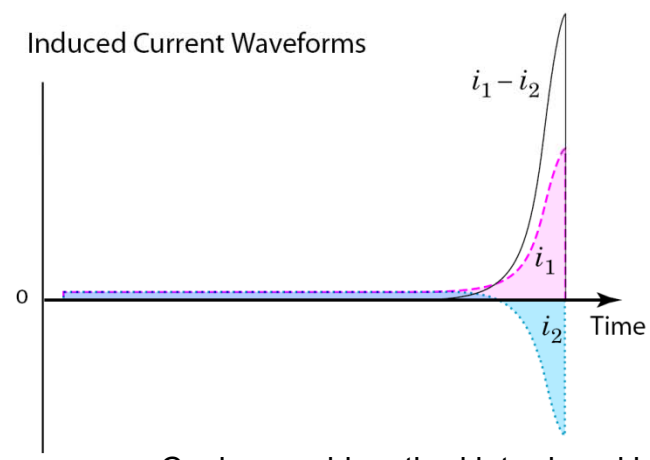
$$i(t)/i_m = 1/\left(1 + t/t_0\right)$$

Coplanar Grid (Strip) Readout: *Difference signal $Q_1 - Q_2$ equals signal charge Q independently of the position of interaction*



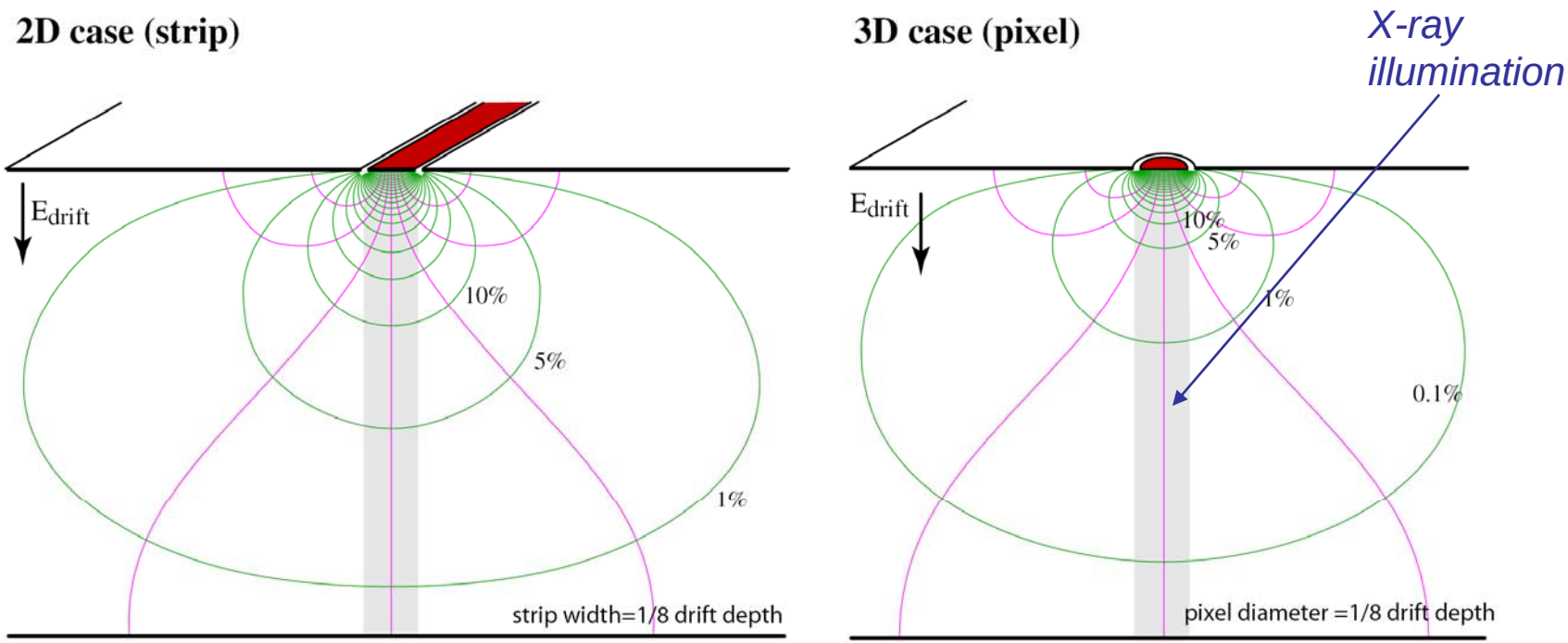


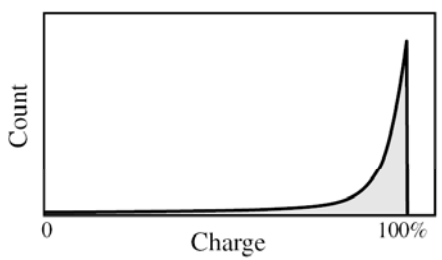




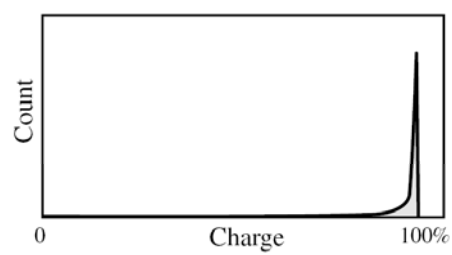
Coplanar grid method introduced by P. Luke, Appl. Phys. Lett. 65:22 (Nov. 28,1994)

Weighting Potential and Pulse Height Distribution for Strips vs Pixels



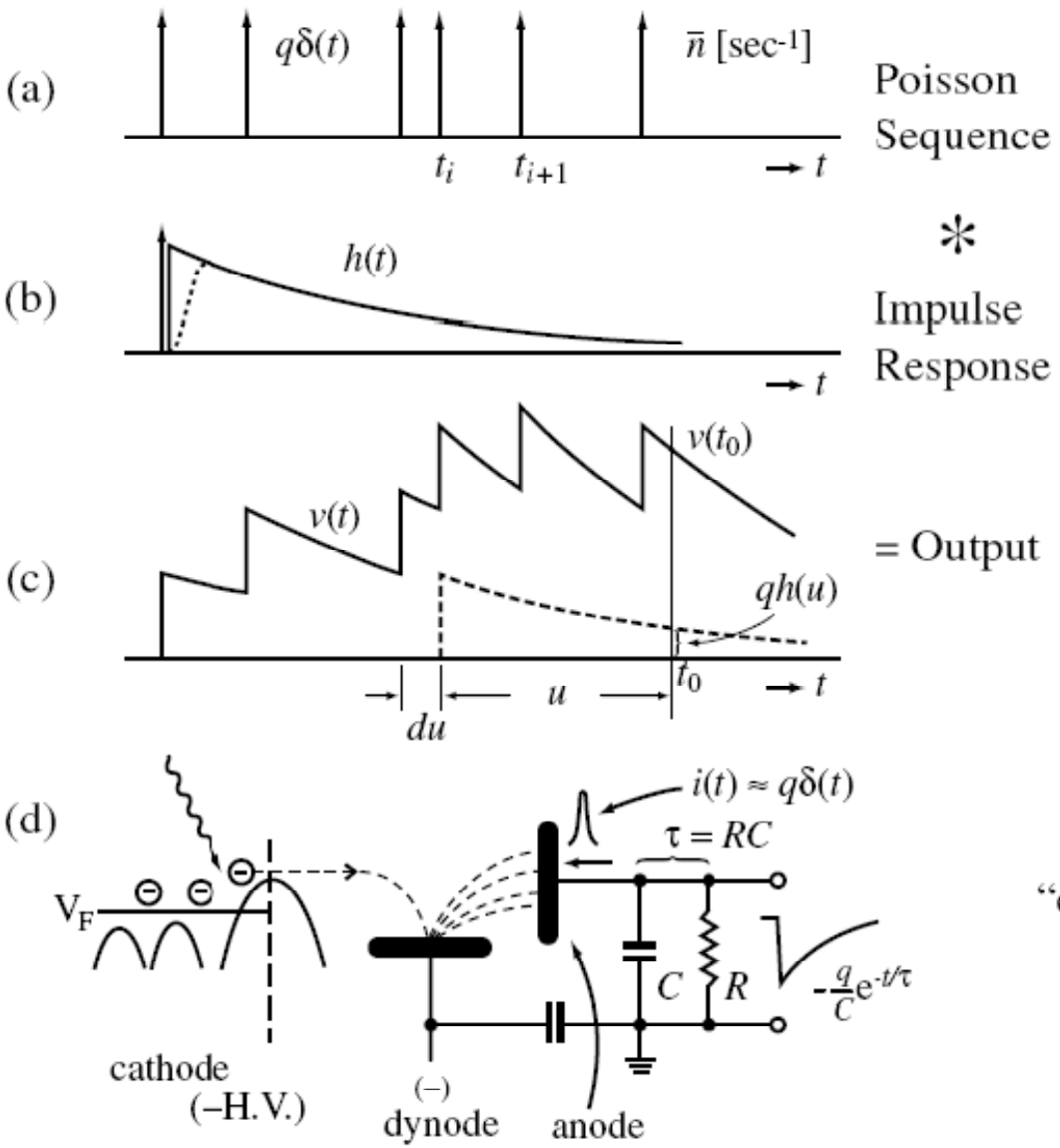


Comparison of their theoretical induced charge pulse height distributions. Assuming an ionization event creates a point charge, and the absorption probability is constant along the depth.



2. Noise mechanisms

Shot Noise, “Pileup”



Campbell:

$$\overline{v^2} = \bar{n}\, q^2 \sum_{i=1}^{N} h^2(t_0 - t_i)$$

$$\overline{v^2} = \sigma^2 = \underbrace{\bar{n}\, q^2}_{\text{noise}} \int \underbrace{h^2(u)du}_{\text{system response}}$$

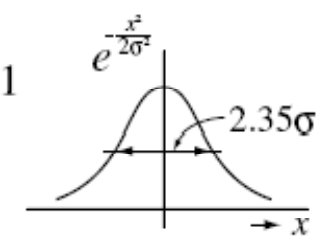
$$\sigma^2 \rightarrow \frac{1}{T_m}\int_0^{T_m} x^2(t)dt$$

Noise Process:

$$\underset{\text{rate}}{\bar{n}}\, q^2 = \bar{n} q \cdot q = \underset{\text{mean current}}{\bar{I}_0} \cdot q$$

“central limit theorem”

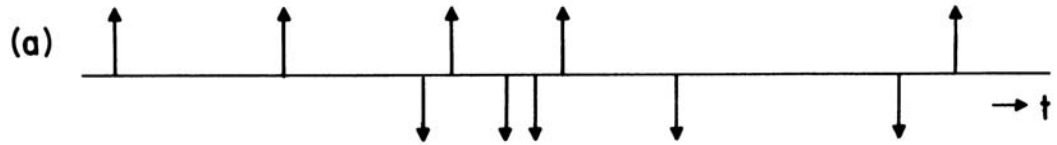
$\longrightarrow$ gaussian for $n\tau \gg 1$



A Model for Generation of Noise Spectra

$$h(t) \overset{F}{\Leftrightarrow} H(j\omega)$$

Random sequence of impulses

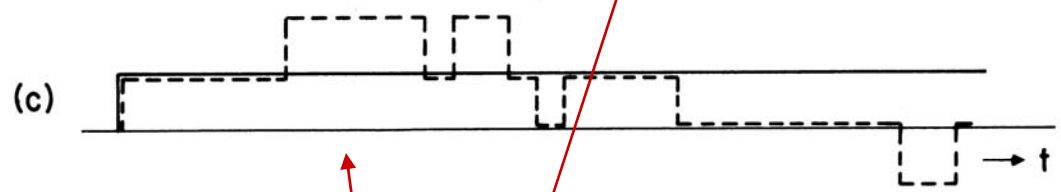


White noise with 1st order band limit (relaxation process)

$$\frac{1}{\tau} e^{-\frac{t}{\tau}} \Leftrightarrow \frac{1}{1 + j\omega\tau}$$

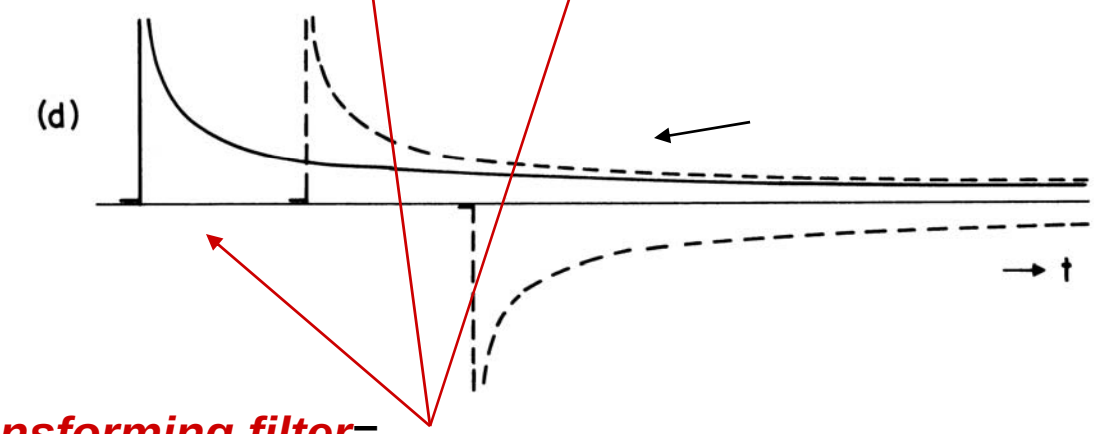
1/f ^{2} noise

(random walk)



$$U(t) \Leftrightarrow 1/j\omega$$

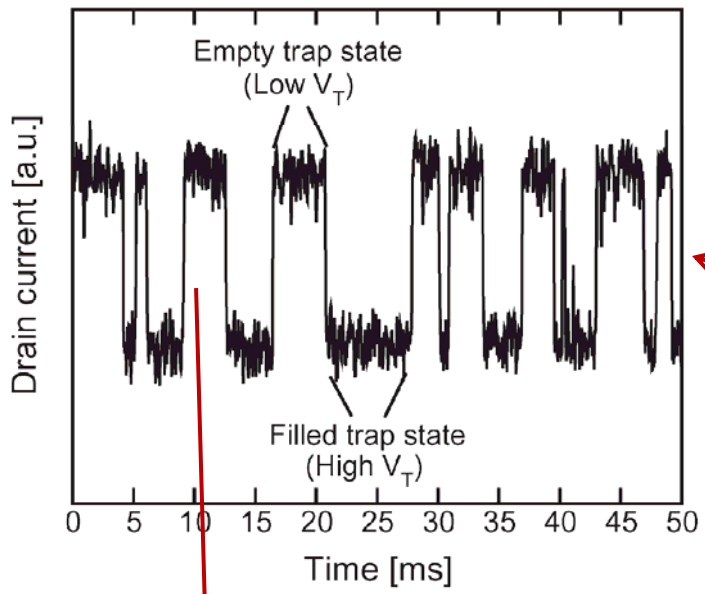
1/f noise



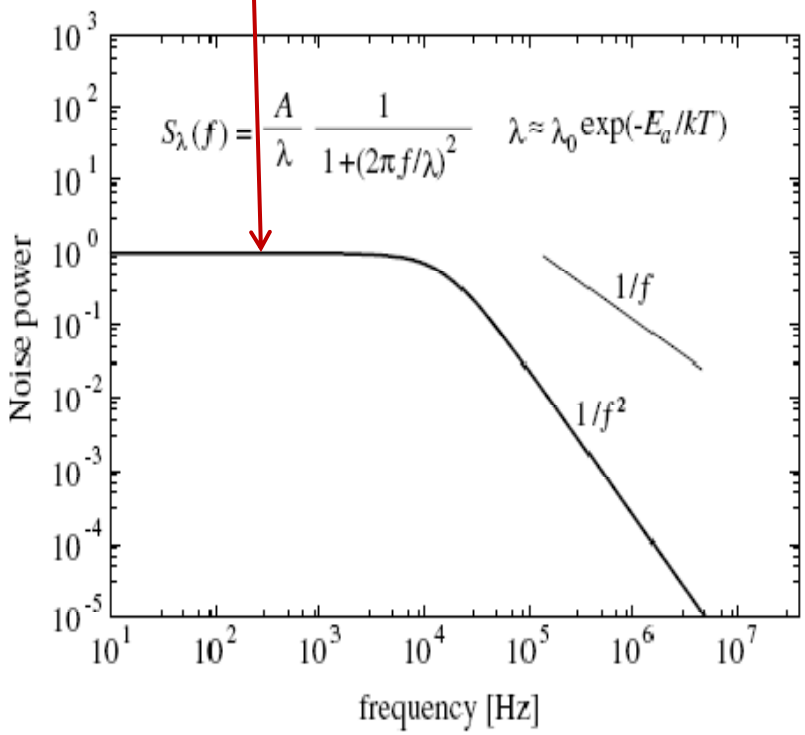
$$U(t)/t^{1/2} \Leftrightarrow 1/(j\omega)^{1/2}$$

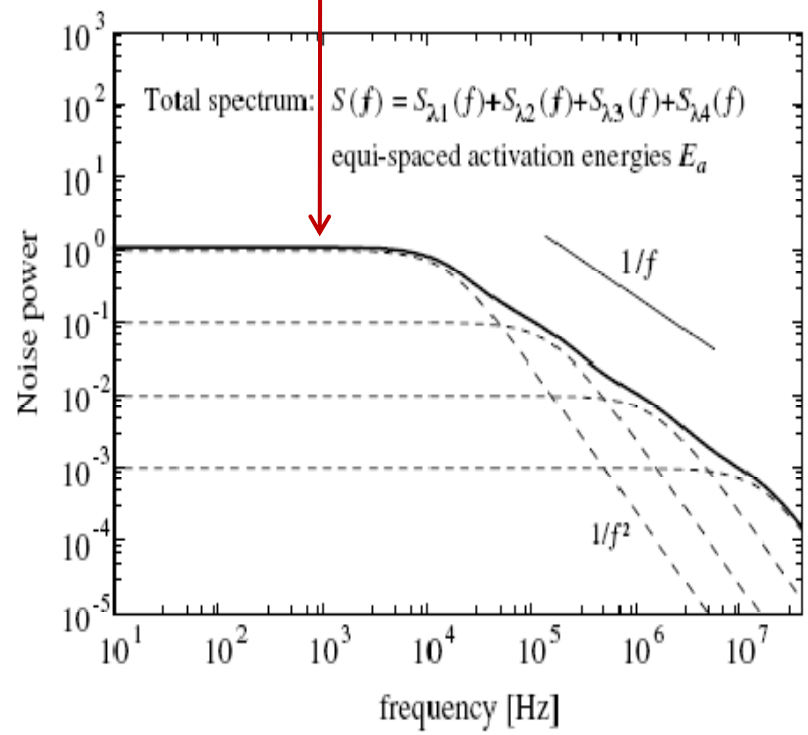
White noise × ***transforming filter***=

Low frequency Noise: Trapping Noise – from RTS (Random Telegraph Noise to 1/f Noise

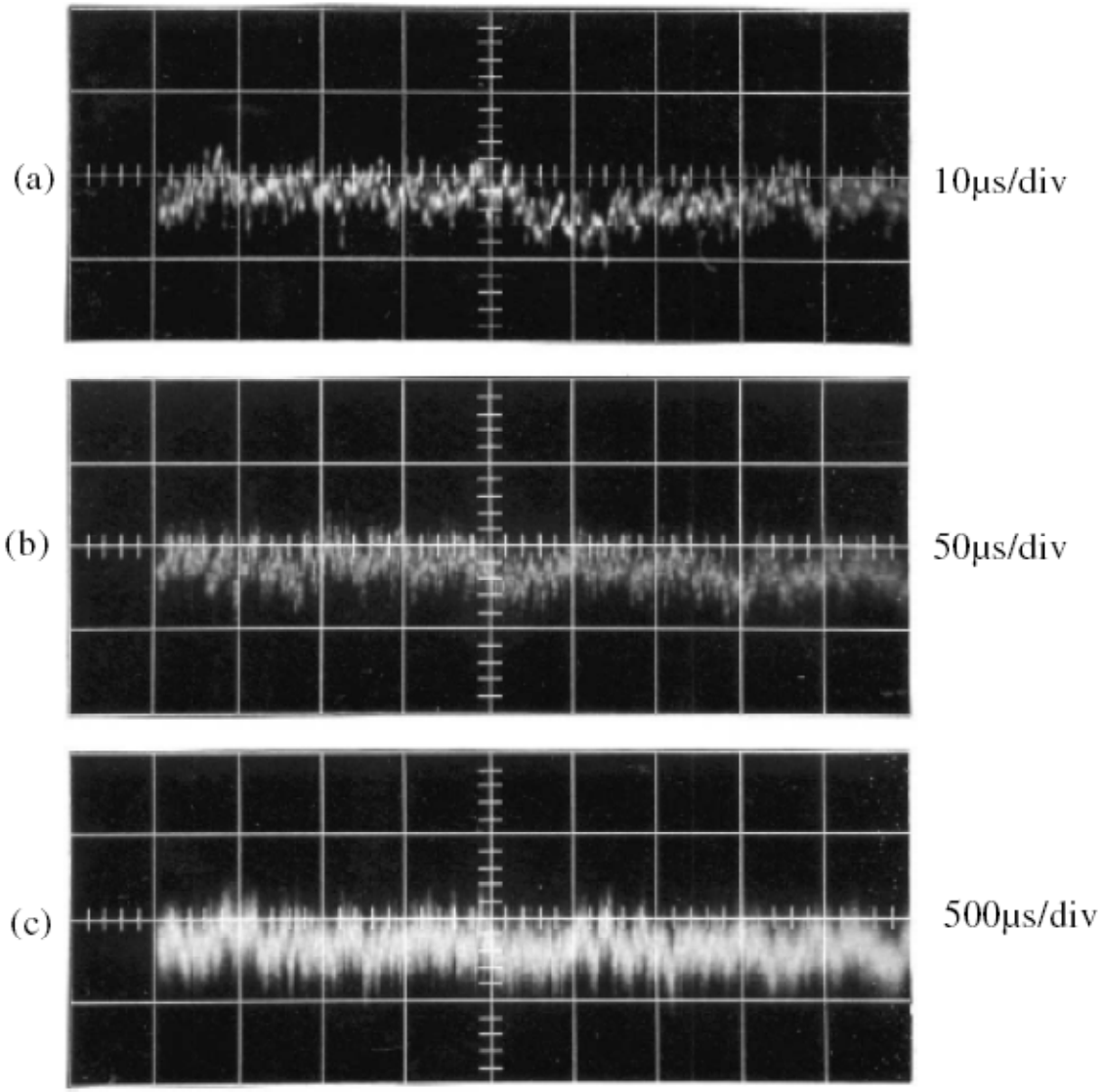


- Single trap → Lorentzian spectrum
- Distribution of traps → 1/f spectrum





“Self-similarity” of 1/|*f*| Noise (scaling invariance)



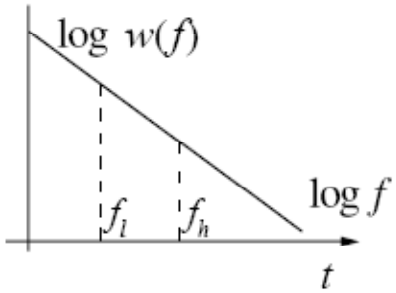
$1/|f|$ noise — different time scales

$$\sigma^2(f_h, f_l) = \int_{f_l}^{f_h} \frac{1}{|f|}\, df = \ln\frac{f_h}{f_l}$$

$$\sigma(f_h, f_l) = \left(\ln\frac{f_h}{f_l}\right)^{1/2}$$

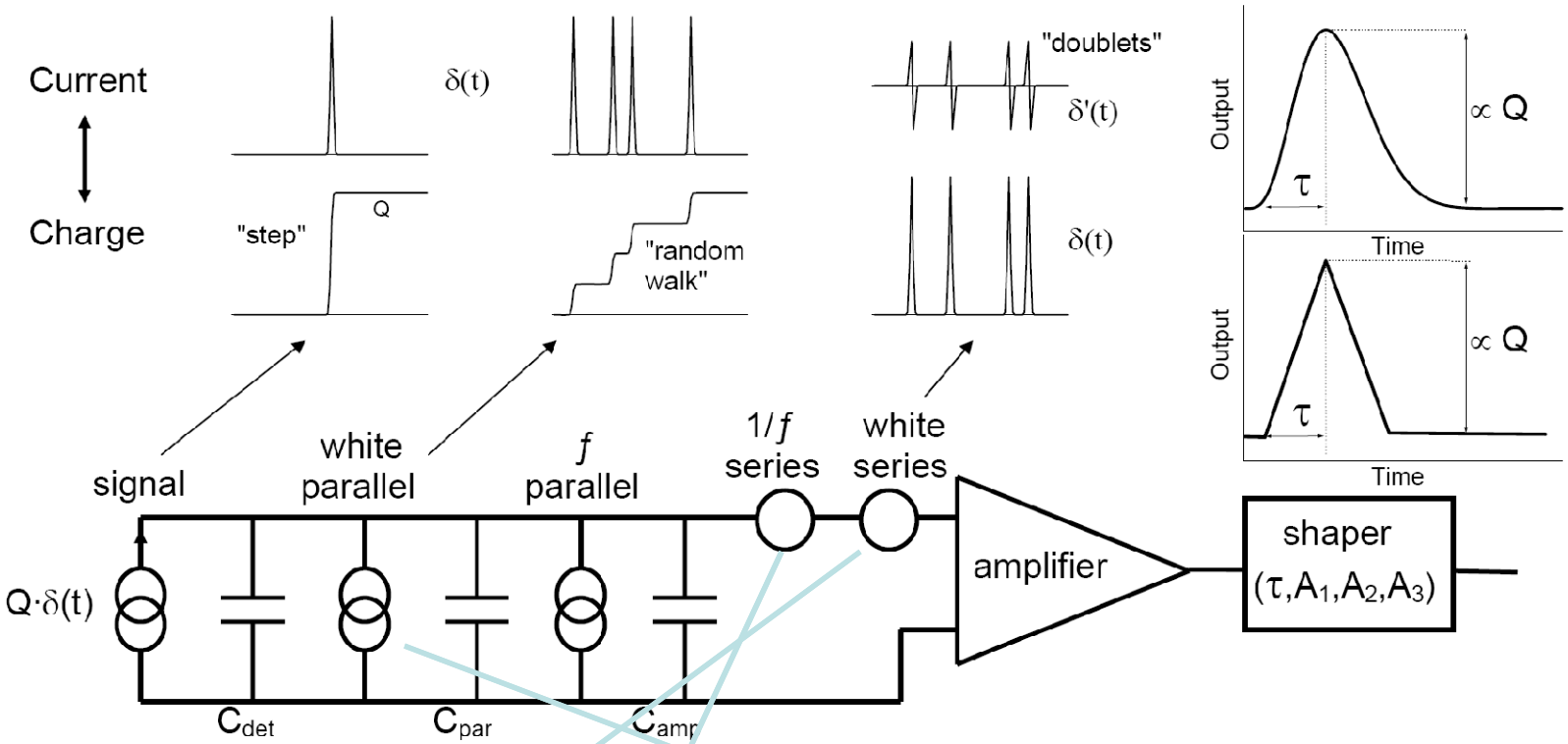
For $\dfrac{f_h}{f_l} = \text{const.}$

$$\underline{\sigma^2(f_h/f_l) = \text{const.}}$$



3. Equivalent noise charge (ENC) calculation

Noise Sources and Spectral Densities in Charge Amplifiers



$$v_n^2 = e_n^2 + \frac{2\pi A_f}{\omega} + 2q_e I_0 \frac{1}{C_{in}\omega^2}$$

$$i_{neq}^2 = v_n^2 \omega^2 C_{in}^2 = \omega^2 C_{in}^2 e_n^2 + 2\pi C_{in}^2 A_f \omega + 2qI_0$$

Noise spectral density:

Voltage: $e_n \left[nV/Hz^{1/2} \right]$

Current: $I_0 \left[fA, pA, nA, \ldots \right]$

Equivalent Noise Charge (ENC) – Integrals and Coefficients A1, A2, A3

$$ENC^2 = \frac{1}{2\pi}\int_0^\infty i_{neq}^2 \left|H(\omega)\right|^2 d\omega = \frac{1}{2\pi}\int_{-\infty}^\infty \left(\tfrac{1}{2}\right) i_{neq}^2 \left|H(\omega)\right|^2 d\omega$$

$$ENC^2 = \tfrac{1}{2} e_n^2 C_{in}^2 I_1 + \pi C_{in}^2 A_f I_2 + q I_0 I_3 = \tfrac{1}{2} e_n^2 C_{in}^2 \frac{A_1}{\tau} + \pi C_{in}^2 A_f A_2 + q I_0 A_3 \tau$$

$$I_1 = \int_{-\infty}^\infty \left[w'(t)\right]^2 dt = \frac{1}{2\pi}\int_{-\infty}^\infty \left|H(j\omega)\right|^2 \omega^2 d\omega = \frac{A_1}{\tau}$$

Series white

$$I_2 = \int_{-\infty}^\infty \left[w^{(1/2)}\right]^2 dt = \frac{1}{2\pi}\int_{-\infty}^\infty \left|H(j\omega)\right|^2 \omega\, d\omega = A_2$$

1/f

$$I_3 = \int_{-\infty}^\infty \left[w(t)\right]^2 dt = \frac{1}{2\pi}\int_{-\infty}^\infty \left|H(j\omega)\right|^2 d\omega = A_3 \tau$$

Parallel white

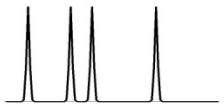
Time domain (weighting function)

Frequency domain (transfer function)

ENC Calculation in Time domain: *Weighting Function Role*

$\delta'(t)\ C_{in}$ white "series" noise

$$ENC_S^2 = \frac{1}{2}\overline{e_n^2}\ C_{in}^2 \overbrace{\int [w'(t)]^2 dt}^{I_1}$$



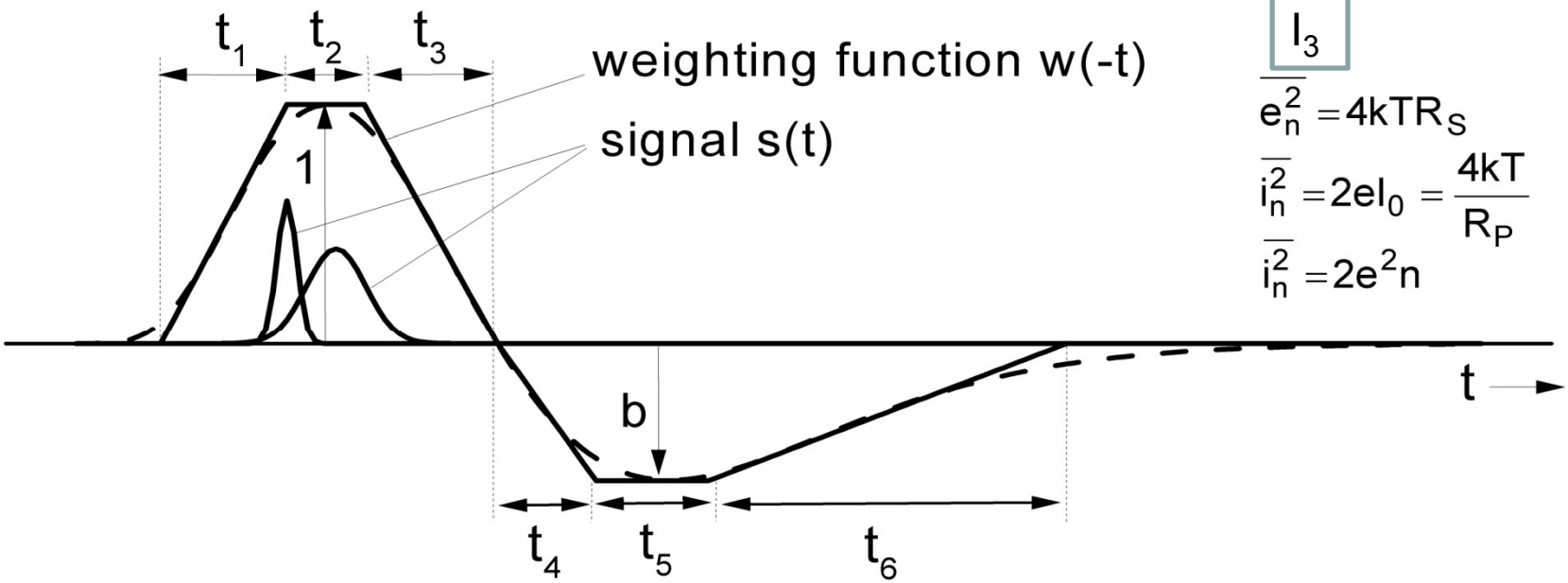
$\delta(t)$ white "parallel" noise

$$ENC_P^2 = \frac{1}{2}\overline{i_n^2} \underbrace{\int [w(t)]^2 dt}_{\boxed{I_3}}$$

$$\overline{e_n^2} = 4kTR_S$$

$$\overline{i_n^2} = 2eI_0 = \frac{4kT}{R_P}$$

$$\overline{i_n^2} = 2e^2 n$$



$$I_1 = \frac{1}{t_1} + 0 + \frac{1}{t_3} + b^2\frac{1}{t_4} + 0 + b^2\frac{1}{t_6}$$

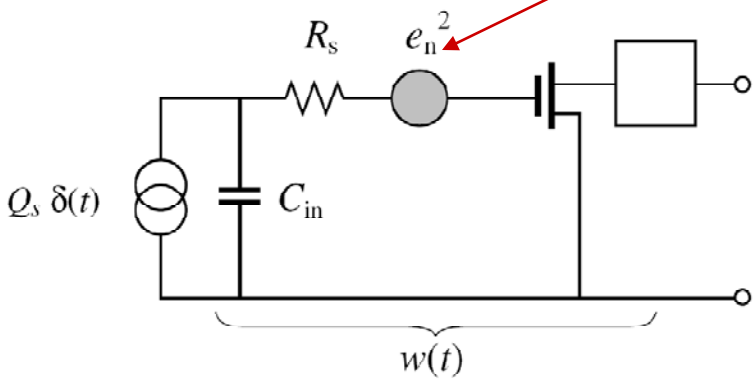
$$\boxed{I_3} = \frac{t_1}{3} + t_2 + \frac{t_3}{3} + b^2\frac{t_4}{3} + b^2 t_5 + b^2\frac{t_6}{3}$$

ENC Calculation in Time Domain

V.R., Ann. Rev. 1988
R. Wilson, Phil. Mag., 1950

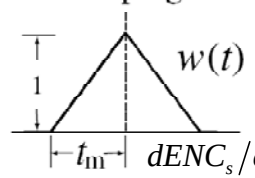
Simple ENC Calculation for ***Series*** White Noise

$$ENC_s^2 = \tfrac{1}{2}\, 4kTR_s C_{in} \bullet I_1$$



$$A_1 = I_1 t_m = 2$$

Shaping:



$$dENC_s / dC_{in} = e_n / t_m^{1/2} \quad [\ ]$$

$$I_1 = \int_{-\infty}^{\infty} w'^2(t)dt = 2/t_m$$

(for 5th order semi-gaussian $I_1 = 2.2$)

$$ENC_s = e_n C_{in} / t_m^{1/2}$$

Noise slope:

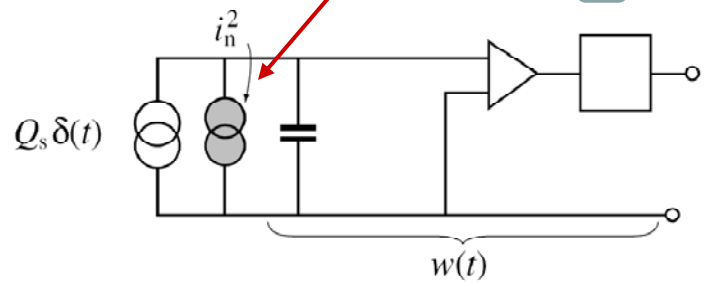
$e_n = 1\ \text{nV/Hz}^{1/2}$
$C_{in} = 1\ \text{pF}$
$t_m = 1\ \mu\text{s}$

$\rightarrow ENC_s \approx 6$ rms e $\rightarrow \Delta ENC_s / \Delta C_{in}$

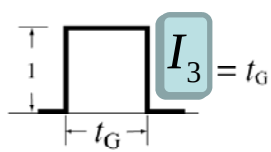
e_n = series noise (voltage) spectral density [V/Hz$^{1/2}$]
$e_n C_{in}$ = series noise (charge) spectral density [e/Hz$^{1/2}$]

Simple ENC Calculation for ***Parallel*** White Noise

$$ENC_p^2 = \tfrac{1}{2}\, \underbrace{2\, eI_o}_{i_n^2} \bullet I_3 \qquad I_3 = \int_{-\infty}^{\infty} w^2(t)\,dt$$



1. Assume a gated integrator for $w(t)$:



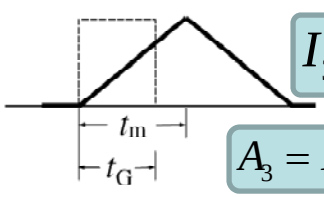
$I_3 = t_G$

$$ENC_p = (eI_o t_G)^{1/2} = (e^2 n t_G)^{1/2} = e(n t_G)^{1/2} = e\, n_G^{1/2}$$

$$ENC_p / e = n_G^{1/2} \quad \text{[rms e]}$$

1 nA in 1μs → $n_G = 6250$ e, $ENC_p = 79$ rms e

2. Triangular weighting function (similar to semi-gaussian)



$I_3 = \frac{2}{3} t_m$

$A_3 = I_3 / t_m = 2/3$

$$ENC_p / e = (2nt_m/3)^{1/2}$$

equal ENC_p for: $t_G = 2t_m/3$

3. Resistor noise = shot noise: $4kT/R_p = 2\, eI_o$

for $R_p I_o = 50$ mV at 293K

Calculation of ENC for 1/f Noise

From slides 16, 17:

$$ENC_f^2 = \pi C_{in}^2 A_f A_2$$

$$A_2 = \int_{-\infty}^{\infty} \left[w^{(1/2)}(t) \right]^2 dt$$

($w^{(1/2)}$=fractional order derivative)

It can be shown that:

$$A_2 \le (A_1 A_3)^{(1/2)}$$

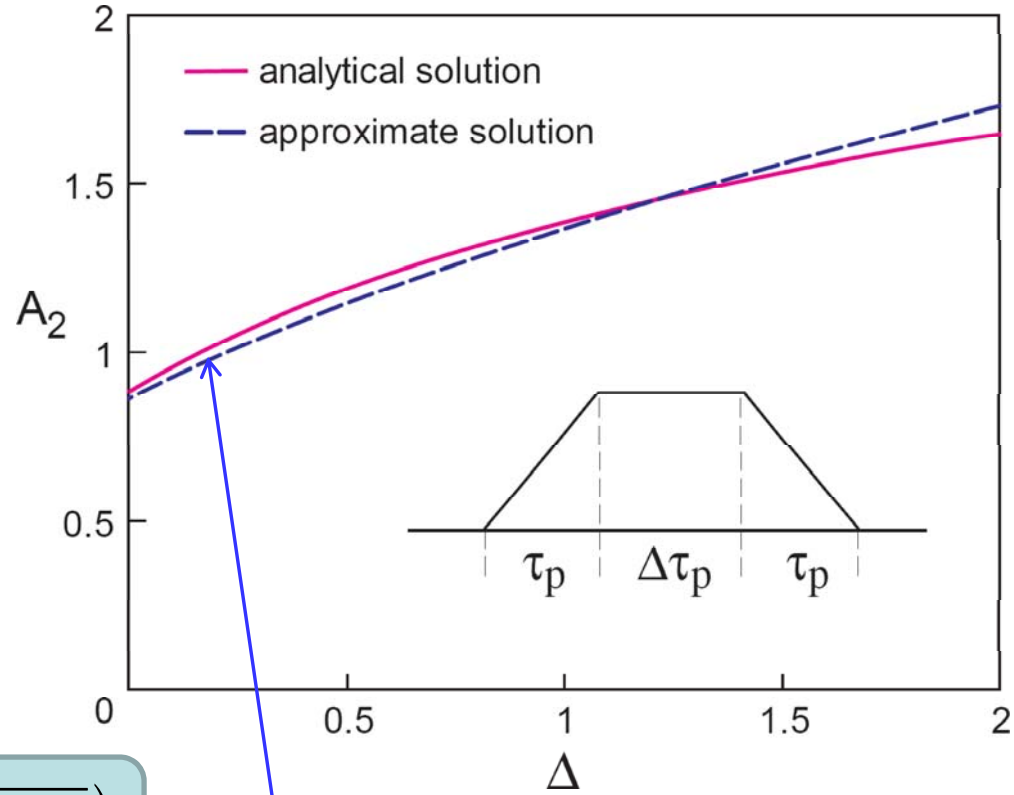
An approximation for practical purposes (*ENC_f to 10%):*

$$A_2 \approx 0.75 (A_1 A_3)^{1/2}$$

$$ENC_f^2 = \pi C_{in}^2 A_f A_2 \approx \pi C_{in}^2 A_f \left(0.75 \sqrt{A_1 A_3} \right)$$

Transistor technology constant (~independent of transistor size) :

$$K_f = A_f C_{gs} \qquad \left[(V^2)(As/V) \right] = [VAs] = [Joule]$$



Trapezoidal weighting function (e.g., dual slope integrator):

$$A_1 = 2$$

$$A_3 = \left(\Delta + 2/3 \right)$$

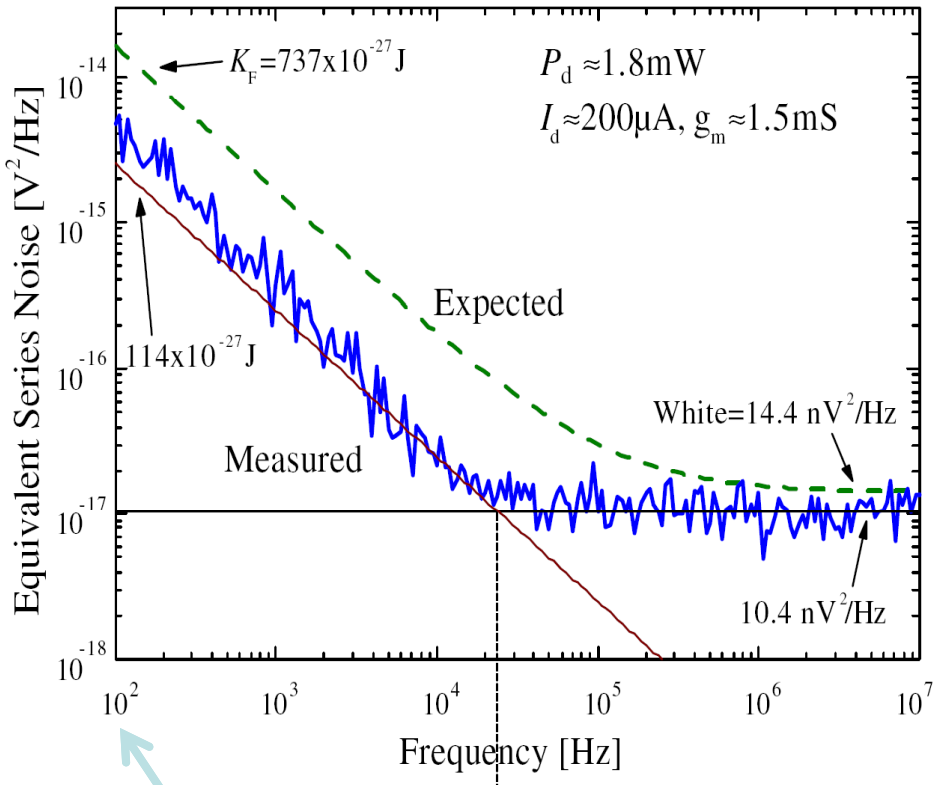
$$A_2 \approx 0.75 \left[2 \left(\Delta + 2/3 \right) \right]^{1/2}$$

Weighting Function Noise Coefficients

Weighting function	Series white A_1	Parallel white A_3	Series *1/f* $A_{2(calc)}$	Series 1/f $A_{2(approx)} \approx 0.75(A_1A_3)^{1/2}$
triangle	2	2/3	0.88	.87
semi-gaussian 4th order	2.04	0.90	1.04	1.01
CR-RC	1.85	1.85	1.18	1.39
trapezoidal Δ=1	2	1.67	1.38	1.37

$$ENC^2 = \tfrac{1}{2} e_n^2 C_{in}^2 \frac{A_1}{\tau} + \pi C_{in}^2 A_f A_2 + q I_0 A_3 \tau$$

***1/f Noise**: corner frequency, "technology constant" K_f and "measurement constant" A_f*



$$f_c \cdot t_{mc} \approx \frac{1}{4} \qquad f_c \approx 25\text{kHz} \qquad t_{mc} \approx 10\mu\text{s}$$

$$A_f = v_n^2 f \approx (3x10^{-15-} V^2/Hz)\cdot(10^2 Hz) = 3x10^{-13} V^2$$

- **Expected white series noise from:**

$$\frac{2}{3}\beta\gamma\frac{4kT}{g_m}$$

$$\begin{cases} \beta \approx 1.2 \text{ (bulk fixed charge } g_{do}/g_m\text{, from simulation)} \\ \gamma \approx 2.2 \text{ (hot electrons, from the literature)} \end{cases}$$

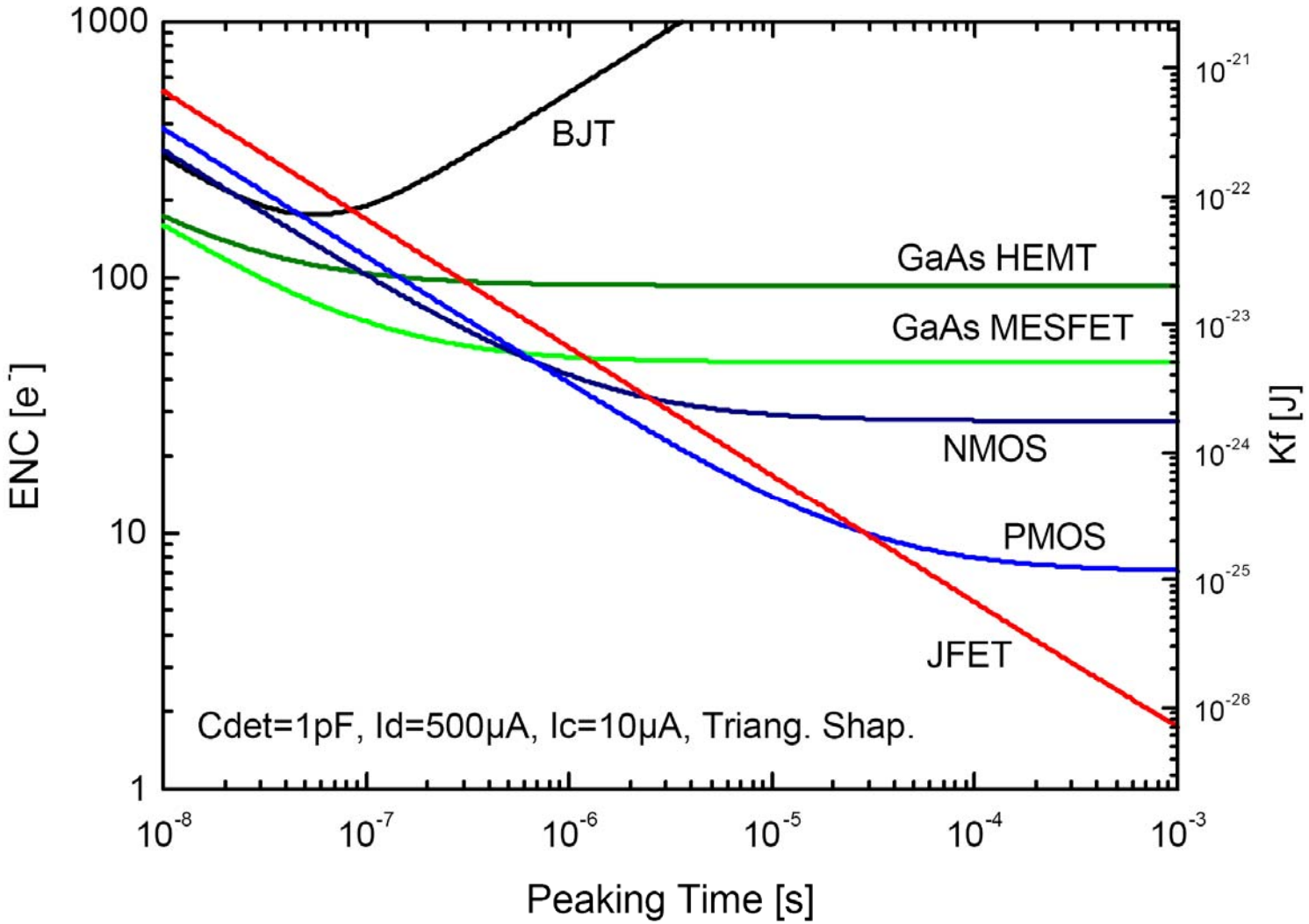
- **Expected 1/*f* series noise from:**

$$\boxed{\frac{K_f}{\underbrace{C_{ox}WLf}_{C_g}} = \frac{A_f}{f} = v_n^2 \quad \left[(rms\,volts)^2\right]}$$

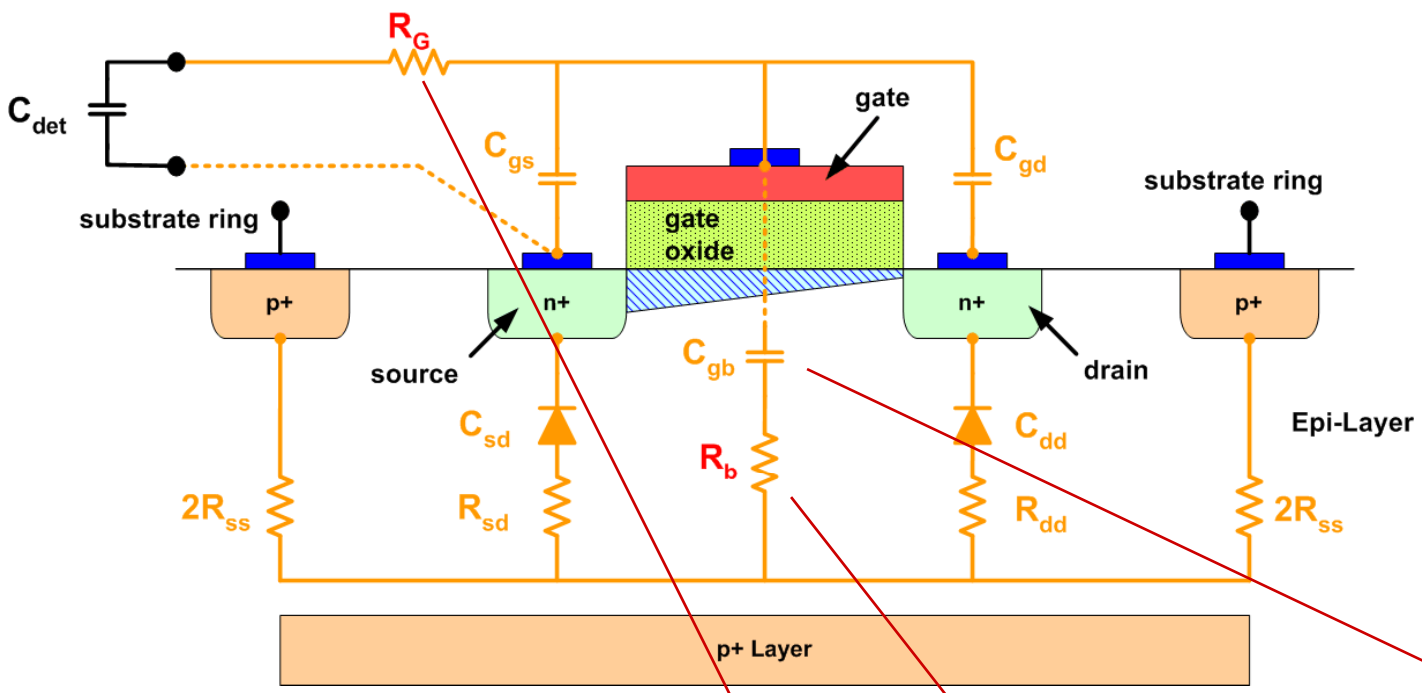
$$\begin{cases} K_{FN} \approx 6.9 \times 10^{-24} \text{ J (from previous meas.)} \\ K_{FP} \approx K_{FN}/10 \text{ (from the literature)} \end{cases}$$

For JFETs $K_F \approx 10^{-27}$

ENC vs Peaking Time for Different Transistor Technologies



Equivalent Series Noise Resistance for Charge Detection (Capacitive Source)



Transconductance=

$$g_{ms} \approx g_{do}$$

$$\frac{R_{eq}}{\gamma / g_{ms}} = 1 + \frac{\delta}{5\gamma}\left(\frac{C_{gs}}{C_{in}}\right)^2 + \frac{1}{\gamma}\left(R_g g_{ms}\right) + \left(R_b g_{mb}\right)\frac{g_{mb}}{g_{ms}} + \frac{1}{\gamma}\left(R_b g_{ms}\right)\left(\frac{C_{gb}}{C_{in}}\right)^2$$

Gate induced

«1

Intrinsic channel noise

Gate resistance

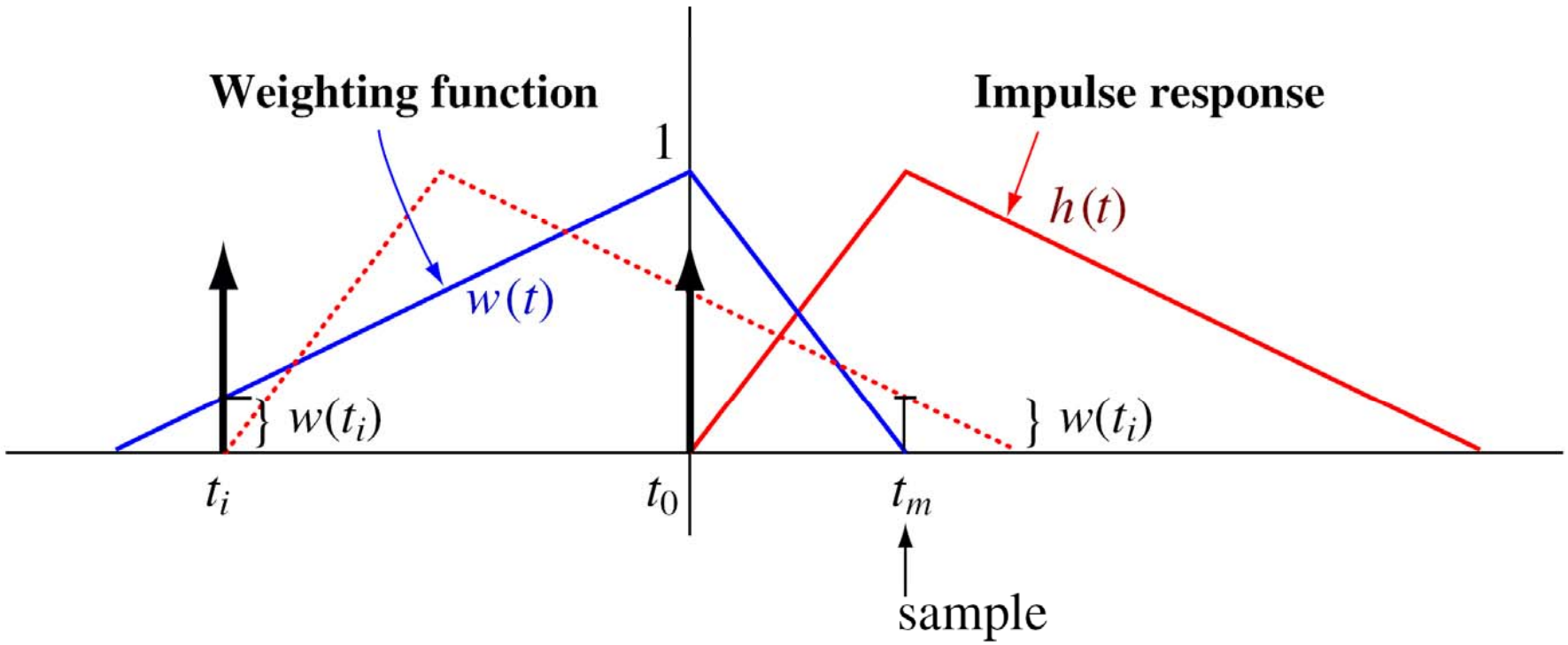
Substrate resistance

Induced into gate (shielded by the inversion layer!?)

Ref.: 9

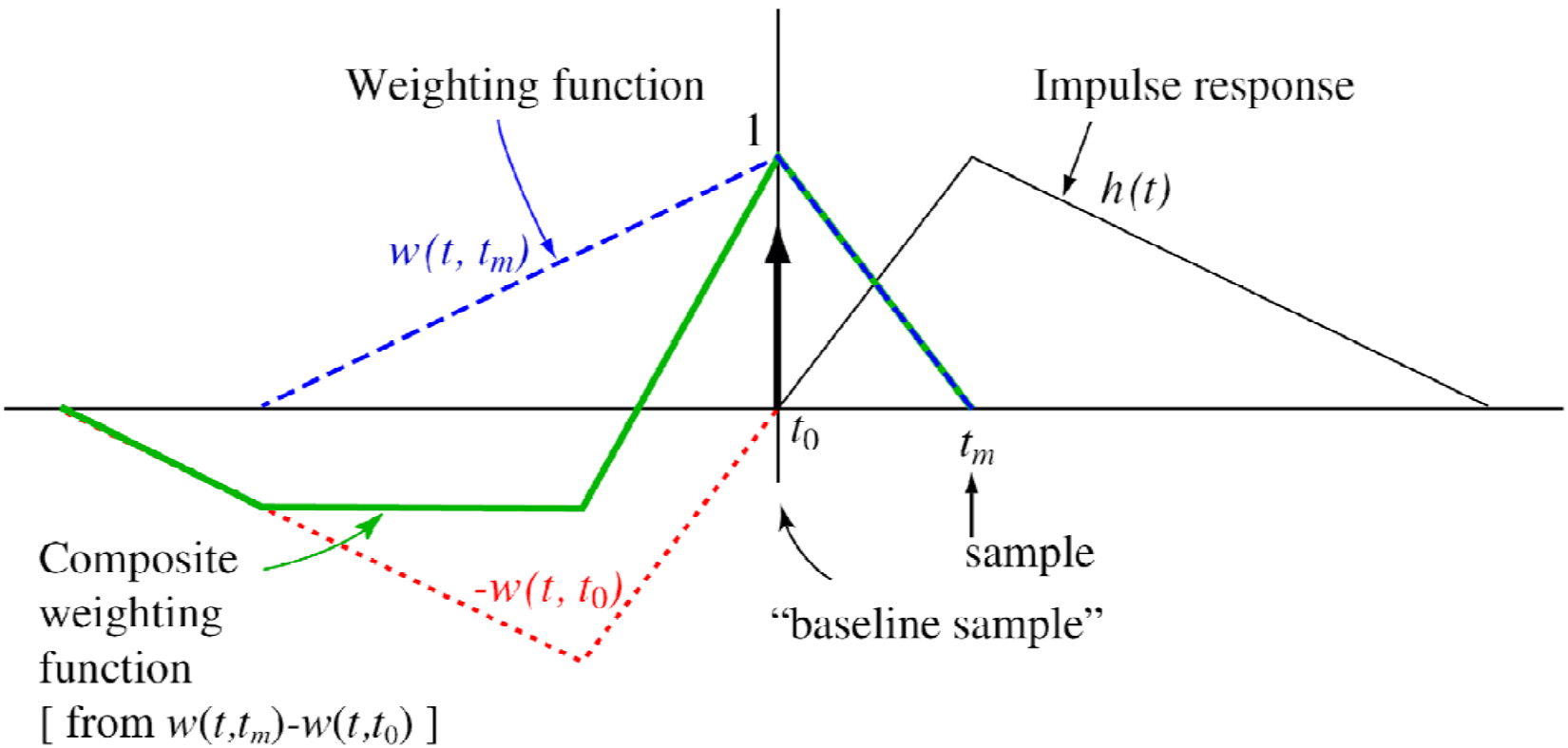
4. Signal processing, i.e., “filtering” or “pulse shaping”

Weighting Function



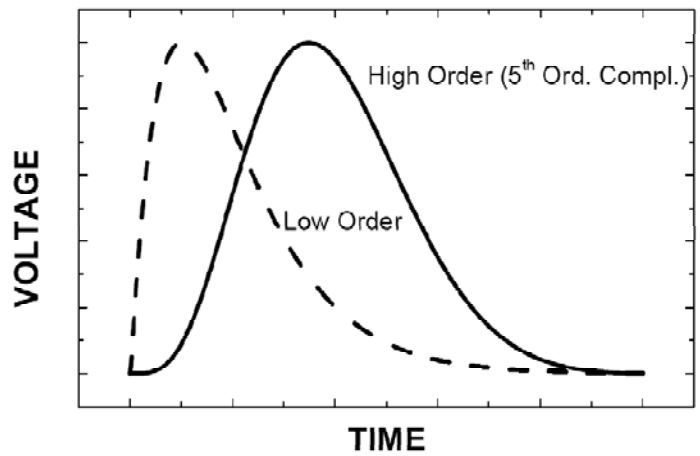
∴ Each sample carries a memory of the past events determined by the impulse response of the system preceding the sampler.

Composite Weighting Function for Correlated Double Sampling



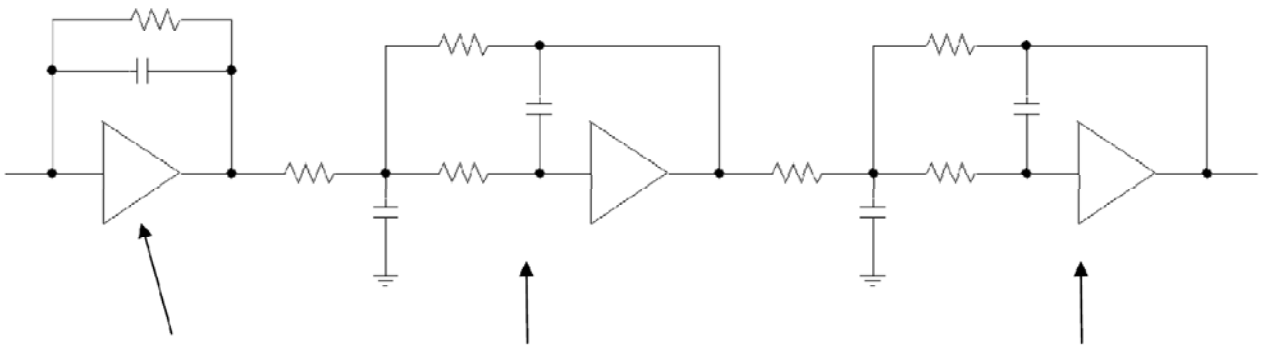
∴ Each sample carries a memory of the past events determined by the impulse response of the system preceding the sampler.

Time Invariant Shaping – Low Order *vs* High Order



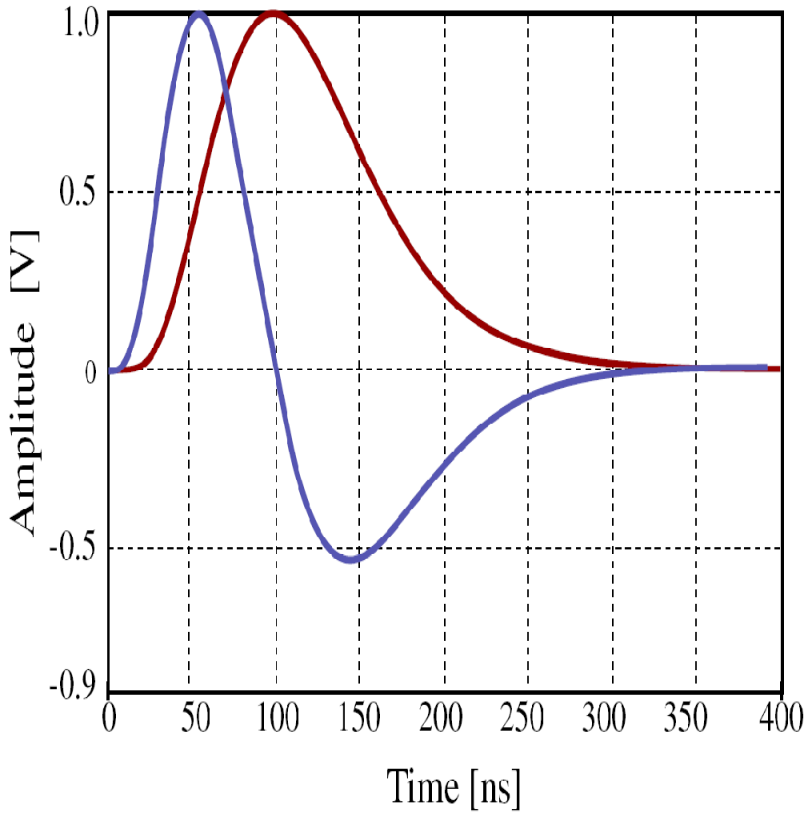
- Better S/N performances
- Lower ballistic deficit

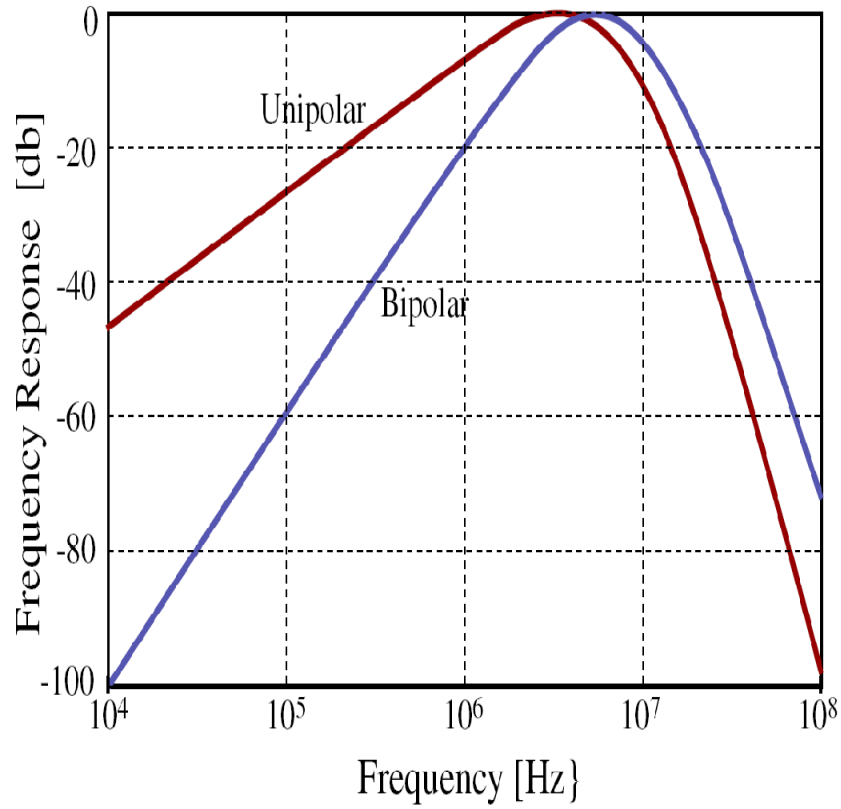
5th Order Complex Semigaussian



$$\frac{a_0}{s+\omega_0} \times \frac{a_1}{s^2+2s\delta_1\omega_1+\omega_1^2} \times \frac{a_2}{s^2+2s\delta_2\omega_2+\omega_2^2}$$

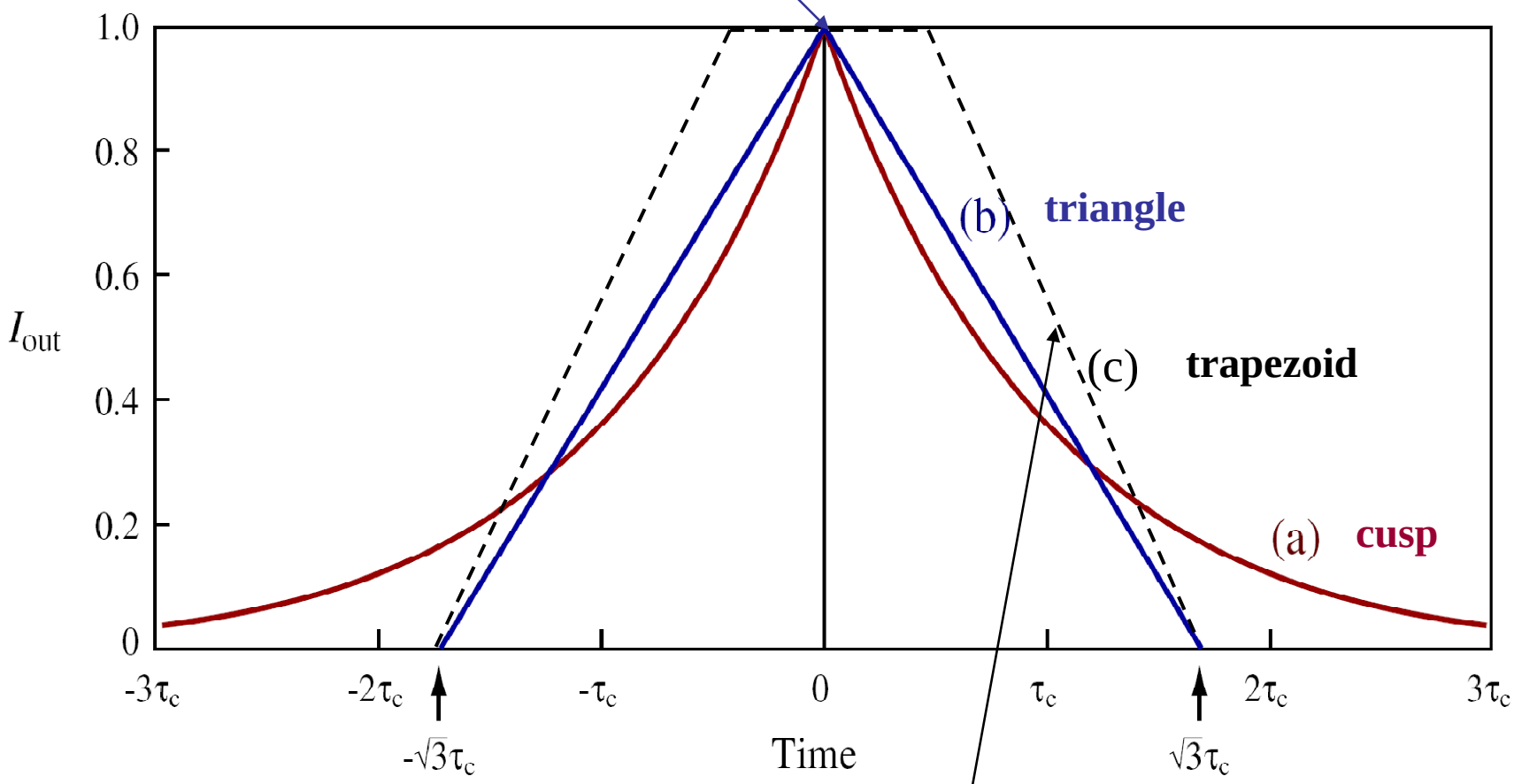
Transfer Functions in Time and Frequency Domains





Optimum Filter for Amplitude Measurement

(delta function signal current)

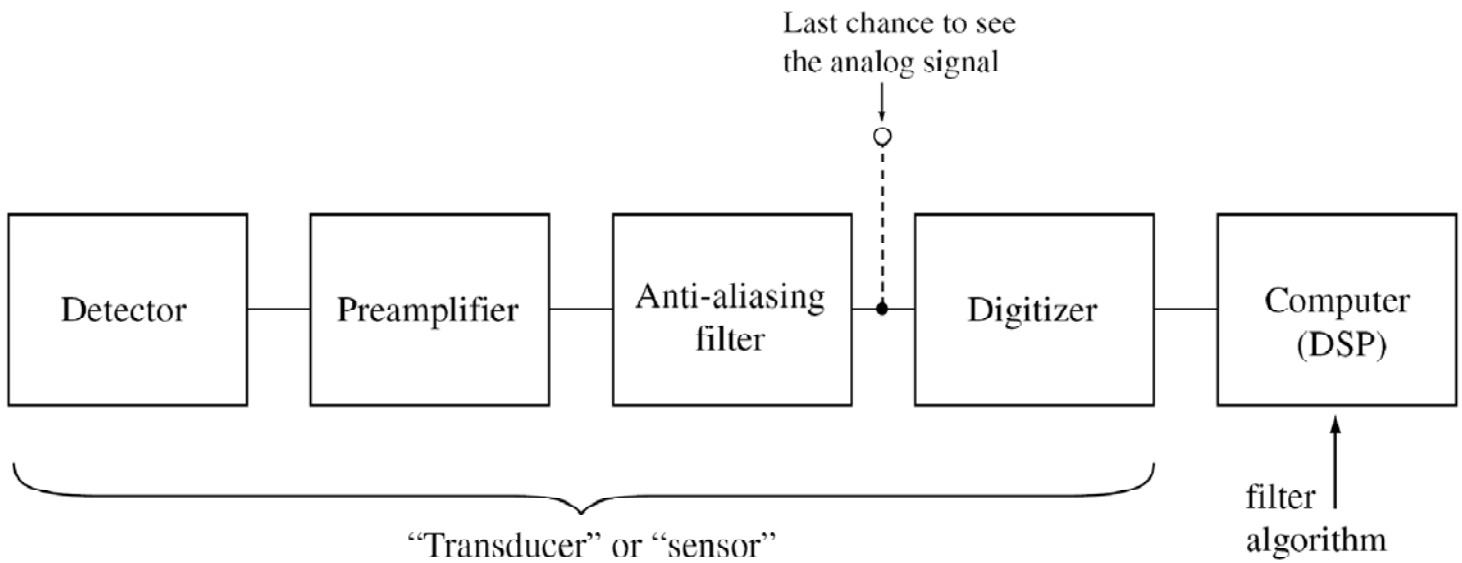


Trapezoid w(t) for reduced ballistic deficit with finite width signal current

τ_c= noise corner time constant

Sampling and Digital Signal Processing - Synchronous (Correlated) and Asynchronous (Uncorrelated)

Sampling System

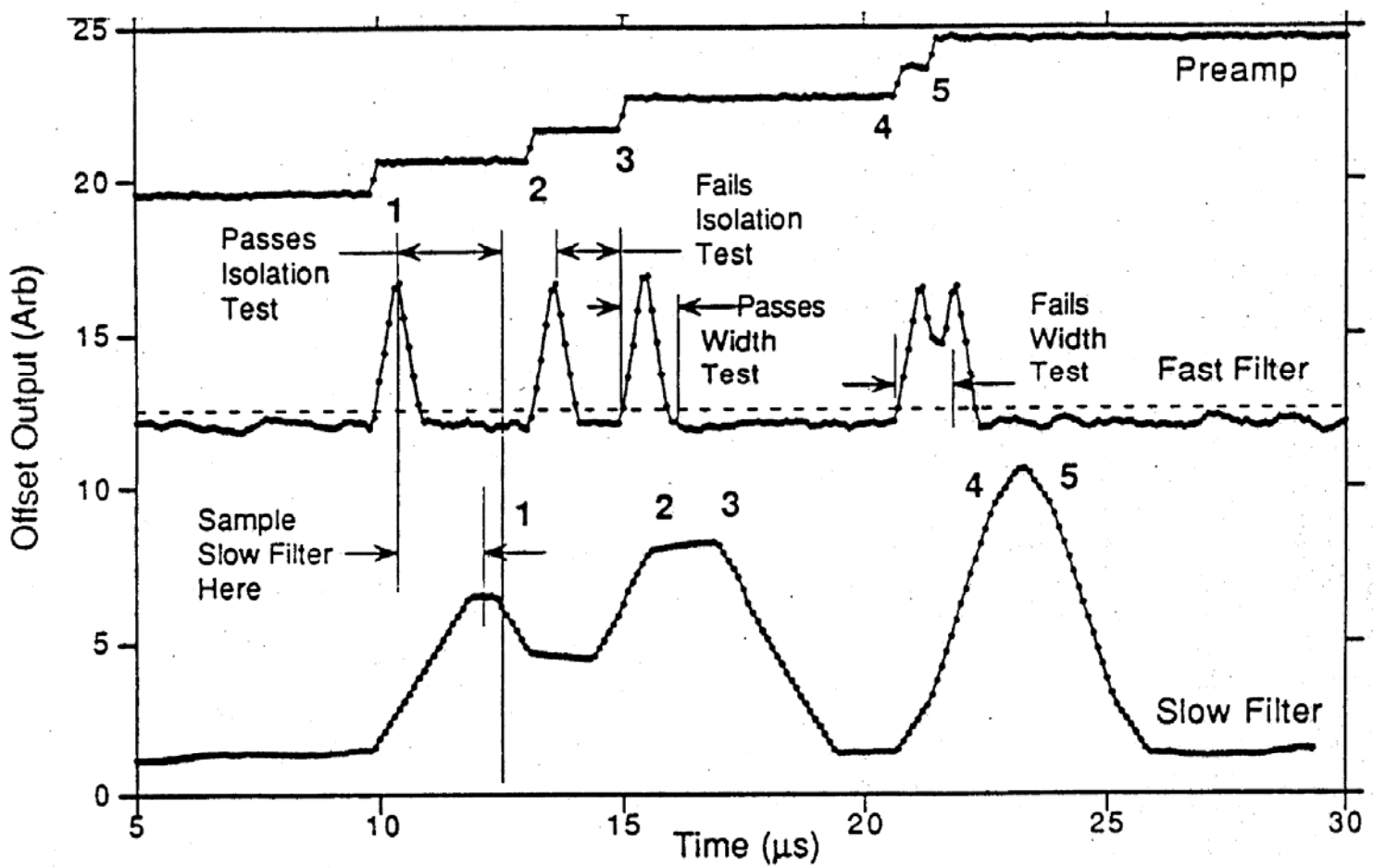


Uncorrelated waveform recording and processing

Note: Long tails in detector-preamplifier response
require cancellation or special processing

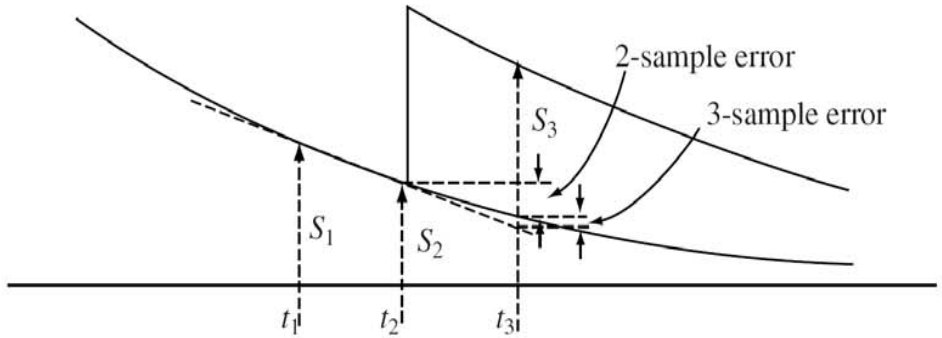
Trapezoidal Weighting Functions
by
Digital (***Uncorrelated***) Signal Processing

From: X-ray Instrumentation Associates,
App. Note 970323-1

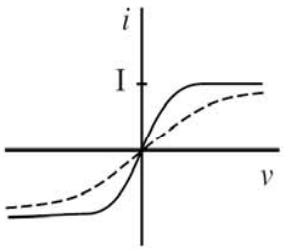


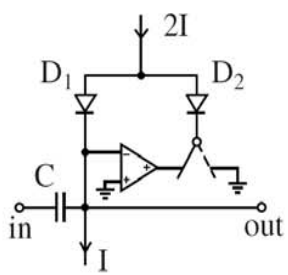
Correlated Sampling by Baseline Restorers (BLRs)

1. BLRs

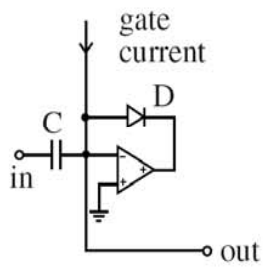


1st order BLRs

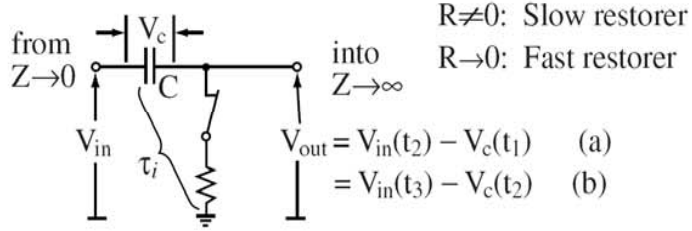




(a) non-linear circuit

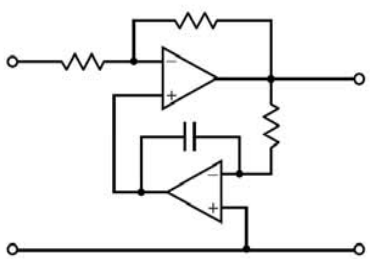


(b) gated BLR

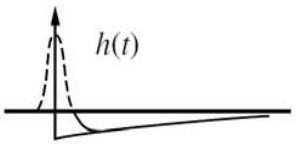


(c) gated BLR

2. Active ac coupling

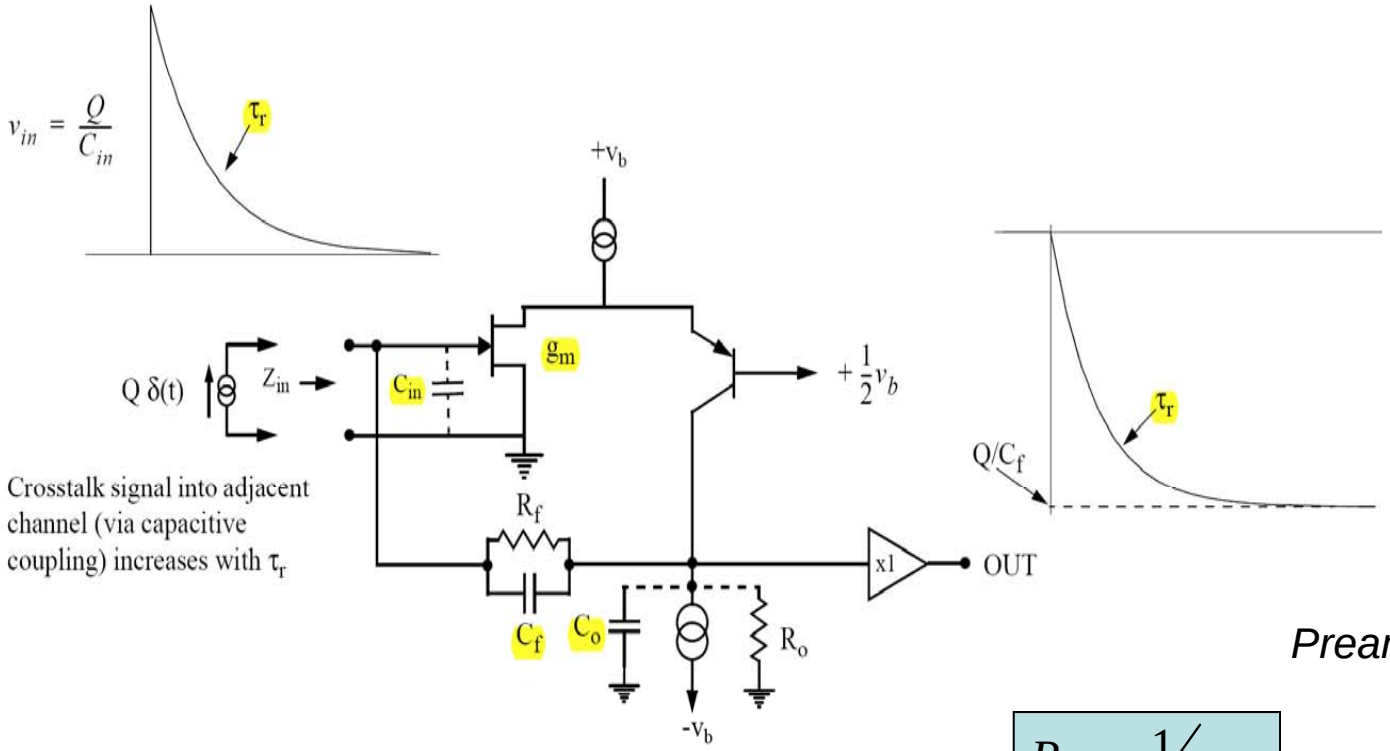


(not a BLR!)



5. Charge amplifier configuration and charge transfer from detector to amplifier

Basic Feedback Preamplifier Configuration



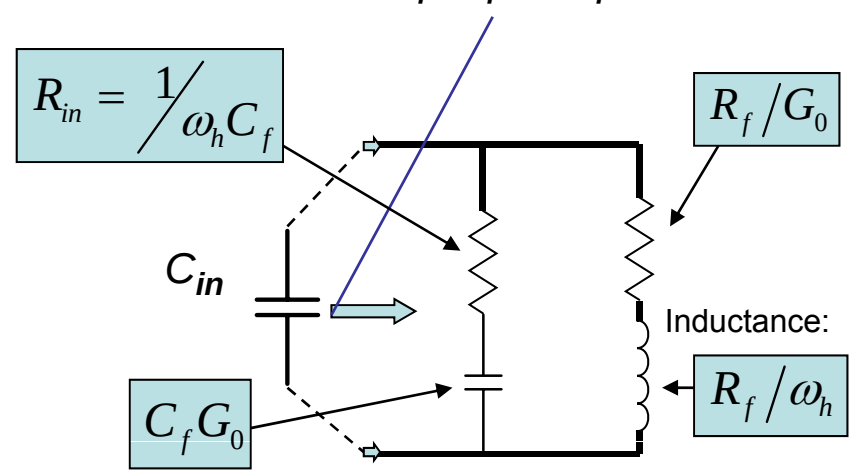
$$G_o = g_m R_o$$

$$\omega_h = g_m / C_o \qquad R_{in} = \frac{C_o}{g_m C_f}$$

$$\omega_L = \frac{1}{C_o R_o} \qquad \tau_r = C_{in} R_{in} = \frac{C_{in}}{C_f\,\omega_h} = \frac{1}{g_m} \cdot \frac{C_{in} C_0}{C_f}$$

Rise t.c.

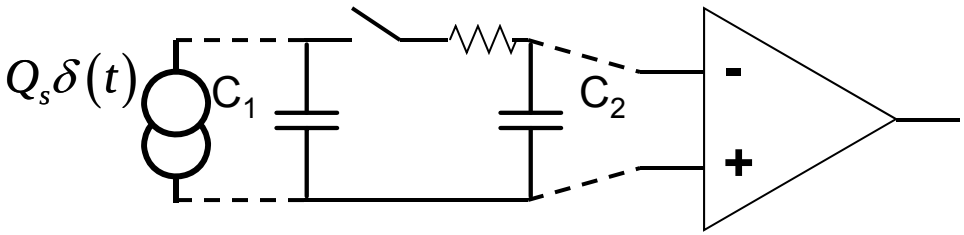
Preamp input eq. circuit



Aperiodic response for $\rightarrow$ $R_f \geq 2 \frac{1}{\omega_h C_f} \cdot \frac{C_{in}}{C_f}$

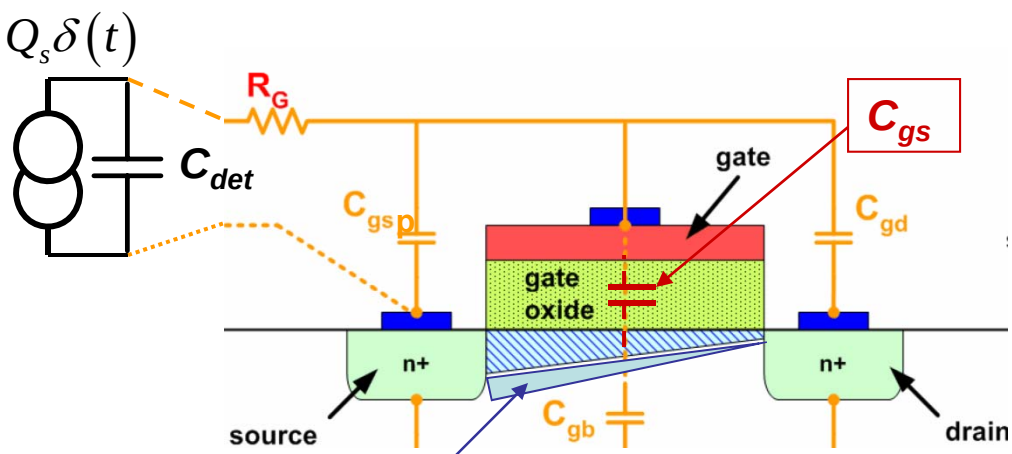
Charge Transfer from Detector to Amplifier

Transfer by *conduction*:



$$Switch\ closed: Q_1 = Q_s \frac{C_1}{C_1 + C_2}; Q_2 = Q_s \frac{C_2}{C_1 + C_2}$$

Q_s *is distributed as the ratio of capacitances*



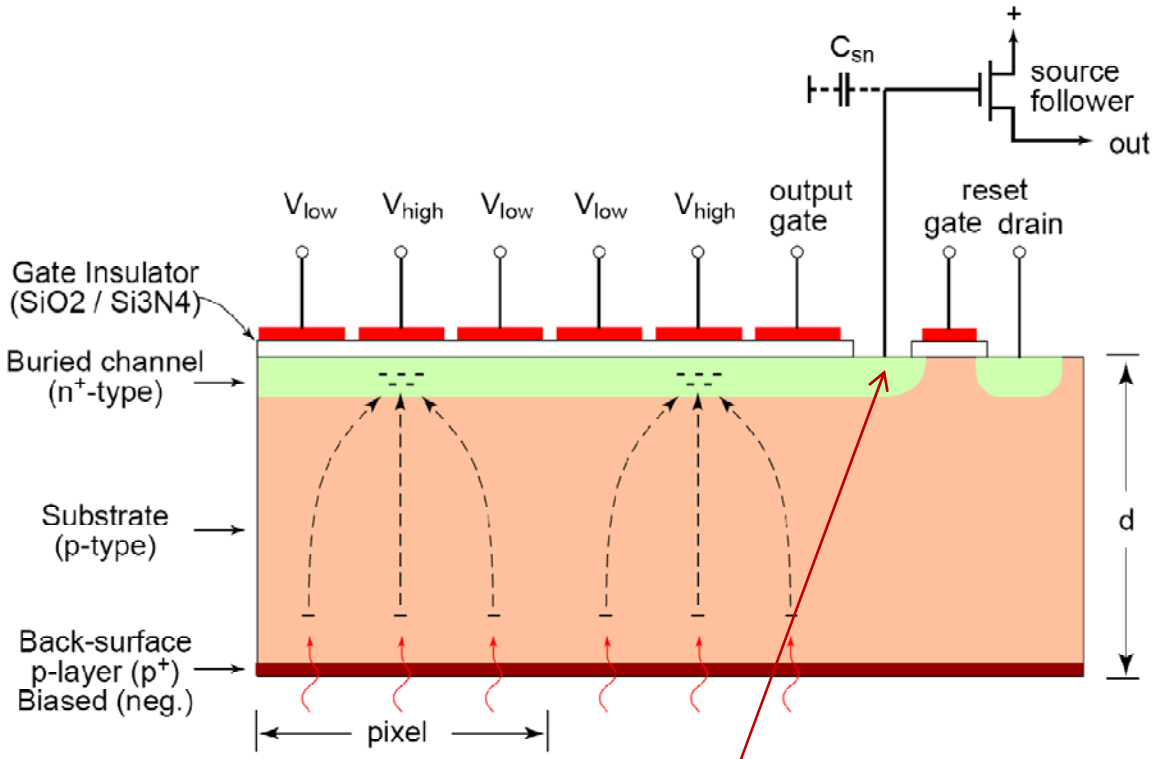
C_{gs} *vs* C_{det} *matching needed!*

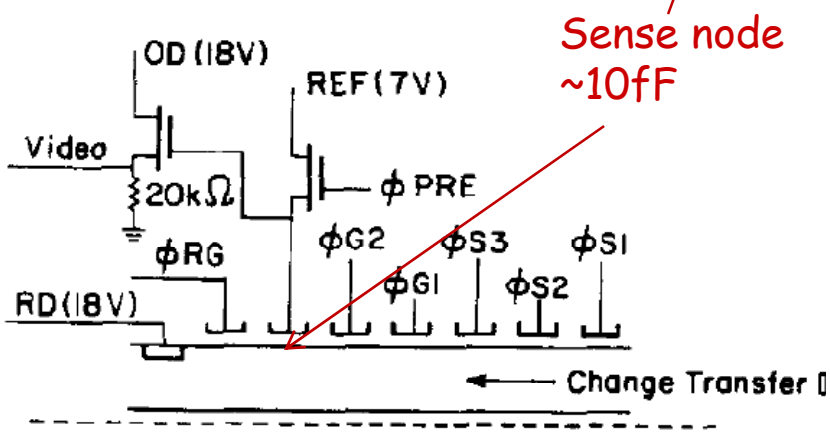
"useful" charge: $\Delta Q_c = Q_s \Big/ \left[1 + \left(C_{det} + C_{gsp} + C_{ds}\right)/C_{gs}\right]$

τ_e = *electron transit time through the channel*

$$\Delta Q_c / \Delta I_d = \tau_e = C_{gs} \Delta V_{gs} / \Delta I_d = C / g_m \rightarrow C_{gs} = g_m \tau_e$$

CCD Readout



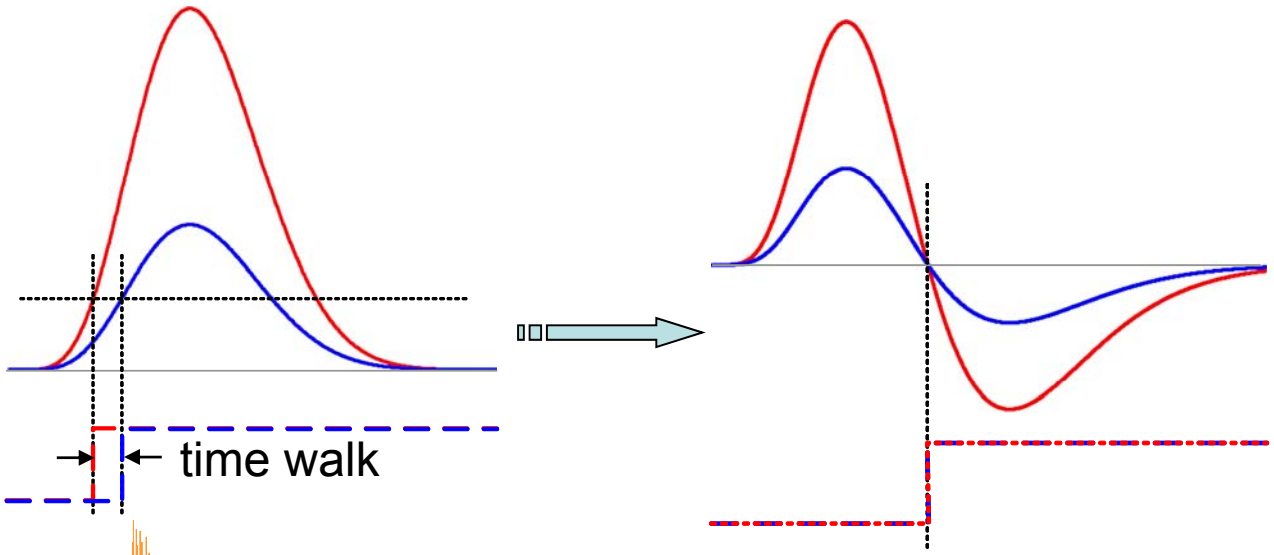


Charge can be shifted back and forth ***nondestructively*** in the buried channel past the sense node allowing for ***multiple measurements***.

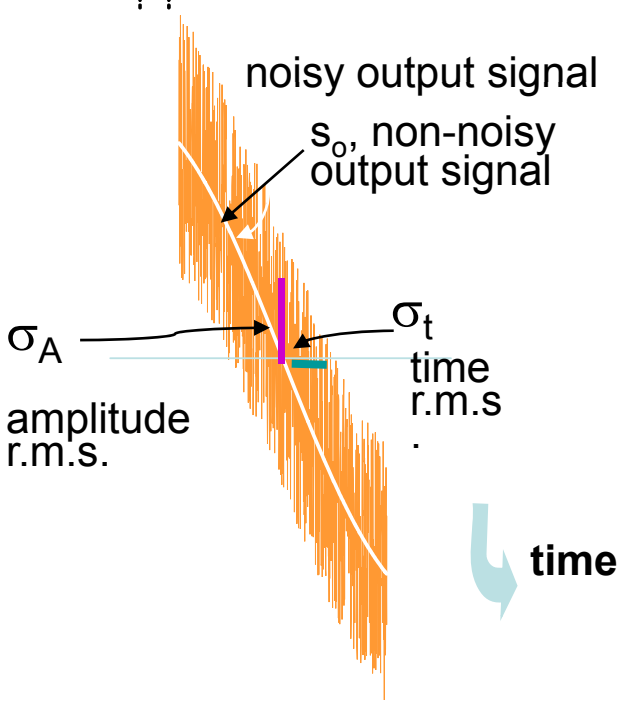
6. Timing measurements

Time Measurements

We want to measure the arrival time of the signal pulse



as time information we choose the 0-crossing time of the output signal



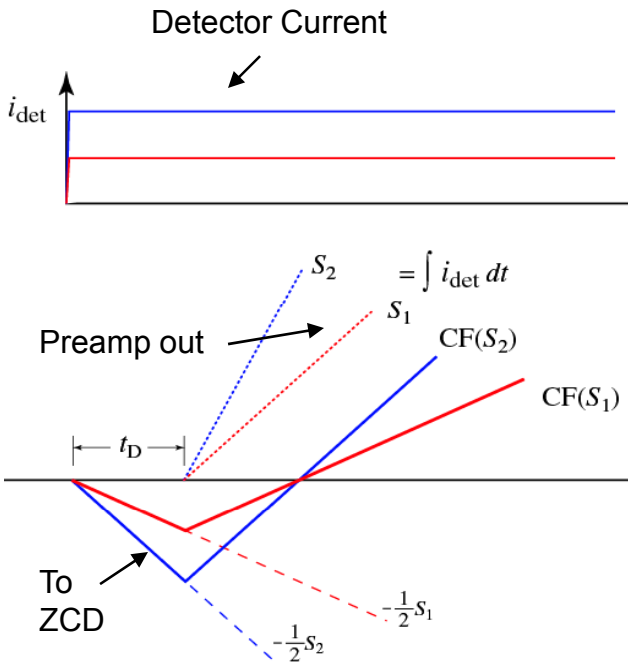
- for simplicity we fix the 0-crossing time in t=0
- due to geometrical considerations:

$$\frac{\sigma_A}{\sigma_t} = \left(\frac{ds_o}{dt}\right)_{t=0} \quad \Rightarrow \quad \boxed{\sigma_t^2 = \frac{\sigma_A^2}{\left(\frac{ds_0}{dt}\right)_{t=0}^2}}$$

time resolution improves as the slope at the 0-crossing increases

From: C. Guazzoni

Constant Fraction Timing of Variable Shape Signals



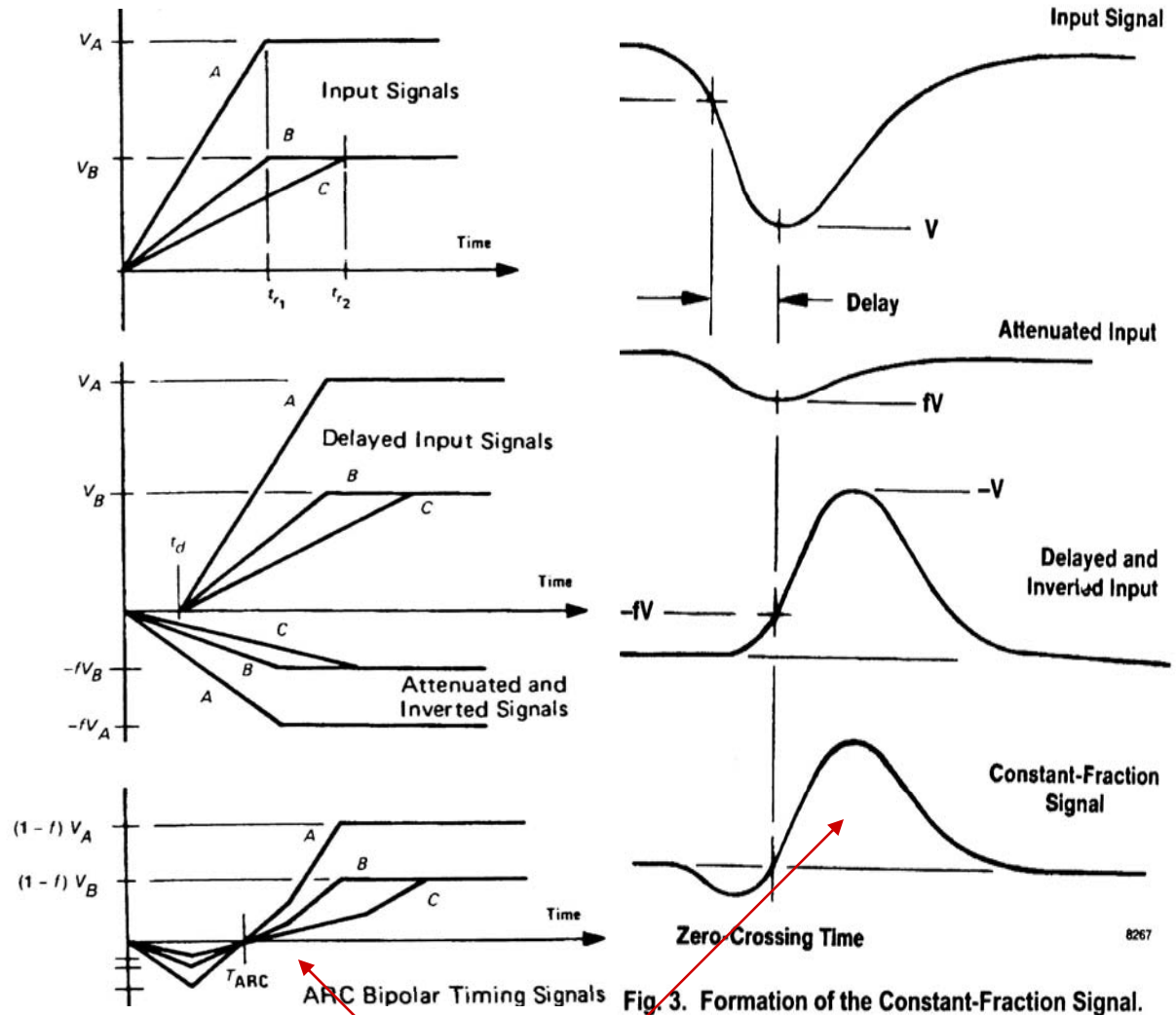


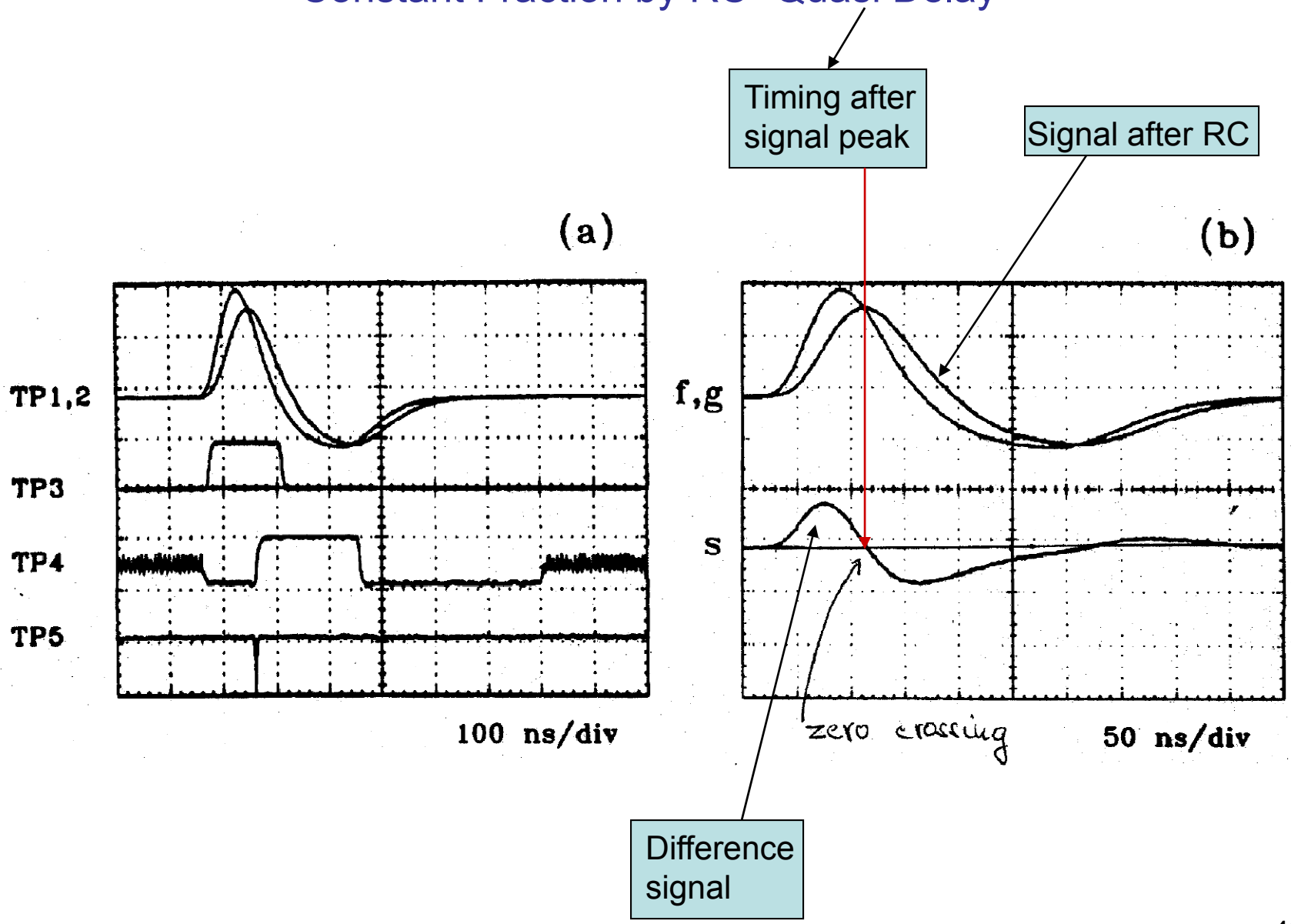
Fig. 3. Formation of the Constant-Fraction Signal.

This method provides ***anti-walk*** timing (constant fraction) on ***early*** part of the signal (short t_D)

Constant fraction signal

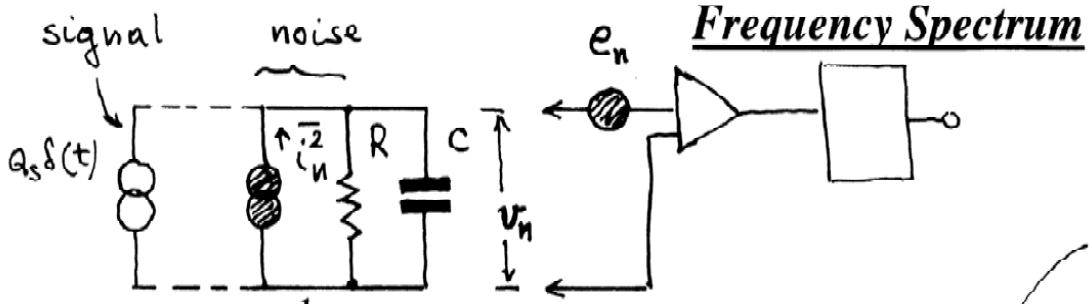
(From: Ortec)

Constant Fraction by RC "Quasi Delay"



7. Fluctuations on a capacitance and charge measurement sensitivity - kTC noise

Thermal Noise, Total Fluctuation: ***kTC noise***

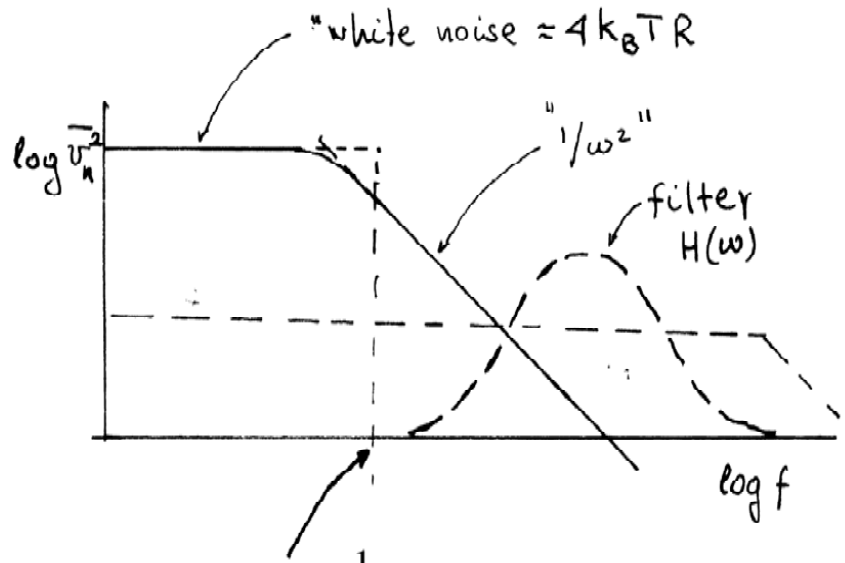


Frequency Spectrum

$$Y = \frac{1}{R} + j\omega C$$

$$\overline{i_n^2} = 4k_BT/R$$

$$\overline{v_n^2} = \overline{i_n^2}\left|Y^{-1}(\omega)\right|^2 = 4k_BTR\frac{1}{1+(\omega CR)^2}$$



Eq. bandwidth $f_h = \dfrac{1}{4RC}$

Total fluctuation:

$$\sigma_v^2 = \int_o^\infty \overline{v_n^2}\,df = \frac{k_BT}{C}$$

$$\sigma_q^2 = k_BTC$$

"independent of R"

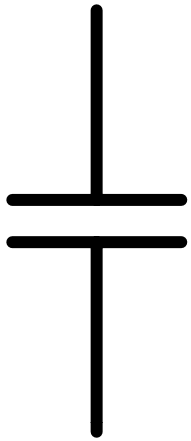
According to equipartion theorem $(1/2)k_BT$ for each degree of freedom →

$$(1/2)k_BT = (1/2)\sigma_q^2/C = (1/2)C\sigma_v^2$$

independent of R

1. C=1pF, R=1GΩ — RC = 1ms, f_h = 250 Hz
2. C=100fF, R=10kΩ — RC=1ns, f_h = 250 MHz

Charge and Voltage Total Fluctuations on Capacitance



Capacitance: $C\ [F]$	***Charge fluctuation:*** $(kTC)^{1/2}/q_e\ [rms\ e]$	***Voltage fluctuation:*** $(kT/C)^{1/2}\ [V]$
1a	**0.4**	**64m**
10a	1.26	20m
100a	4	6.4m
1f	**1.26x10**	**2.0m**
10f	4.0x10	0.64m
100f	1.26×10^2	0.20m
1p	**4.0×10^2**	**64μ**
10p	1.26×10^3	20μ
100p	4×10^3	6.4μ

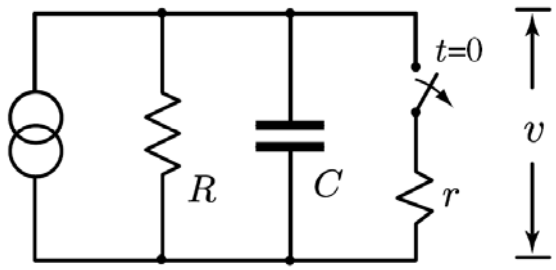
Dynamic range:

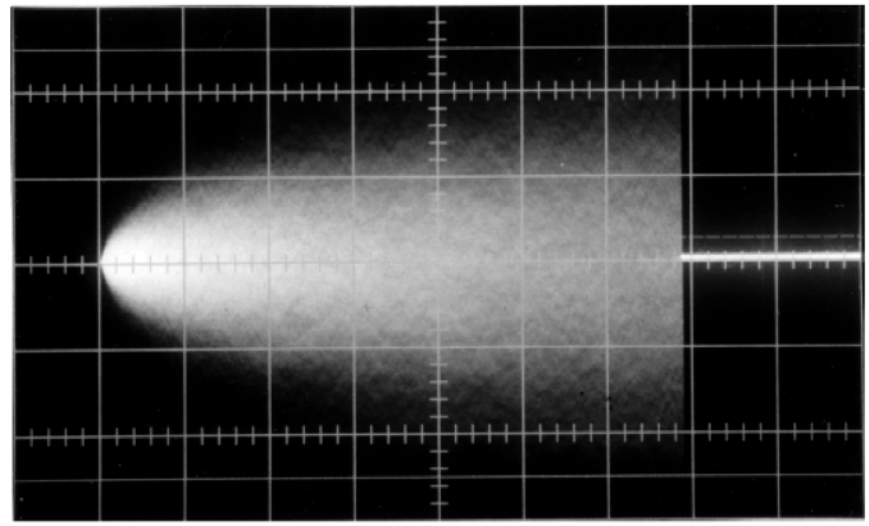
$$500mV/5\sigma \approx 500$$

ENC (if no filtering):
≈126 rms e

Transient Build-up of Noise: ***kTC noise vs time***

$$\overline{i_n^2} = 4k_BT/R$$



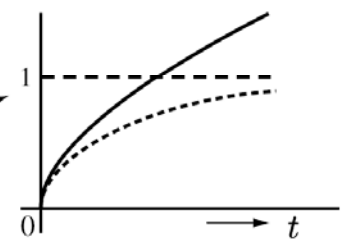


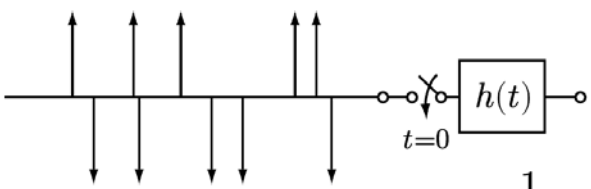
Campbell:

$$\sigma_v^2 = \frac{1}{2}\,\overline{i_n^2}\int_0^t h^2(u)du = \frac{k_BT}{C}(1-e^{-2t/\tau})$$

$$\underline{\sigma_v/(k_BT/C)^{1/2}} = \underline{(1-e^{-2t/\tau})^{1/2}}$$

$$\frac{\sigma\,(t)}{(k_BT/C)^{1/2}}$$





$$h(t) = \frac{1}{C}\,e^{-t/\tau}$$

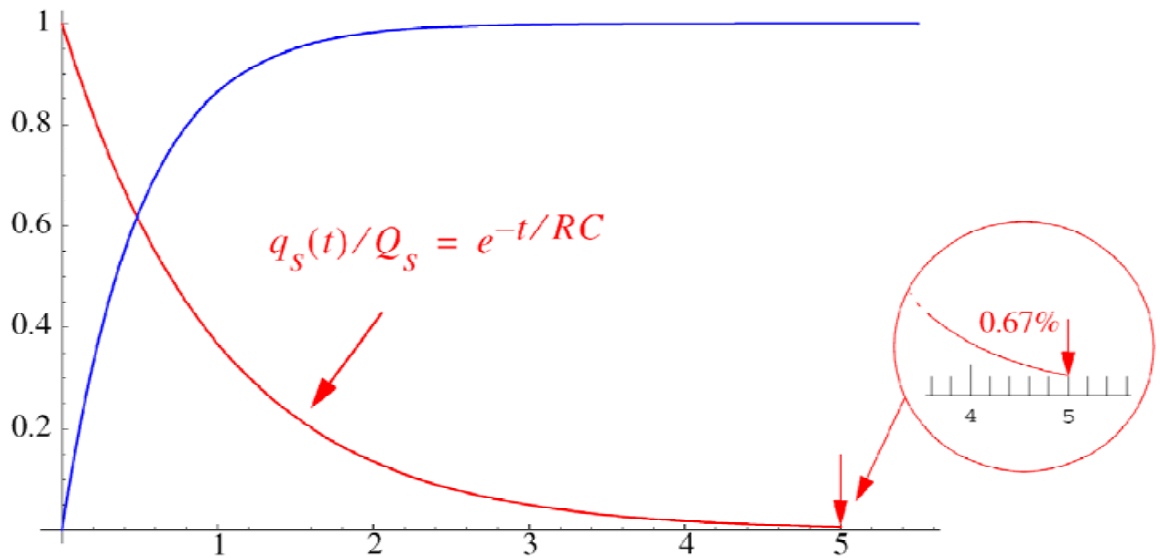
For $t/\tau \ll 1$: $\sigma_v^2 \propto t$ (random walk)

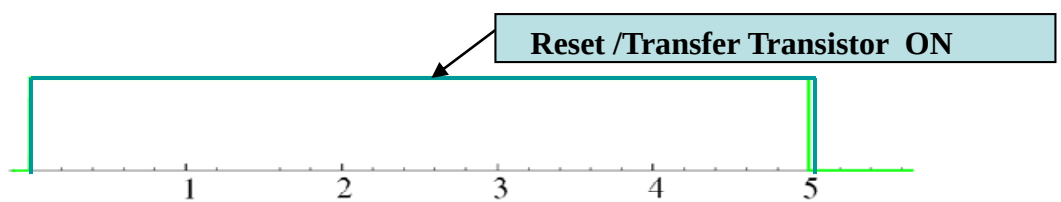
$$\sigma_v \approx (k_BT/C)^{1/2}\cdot(2\frac{t}{\tau})^{1/2}$$

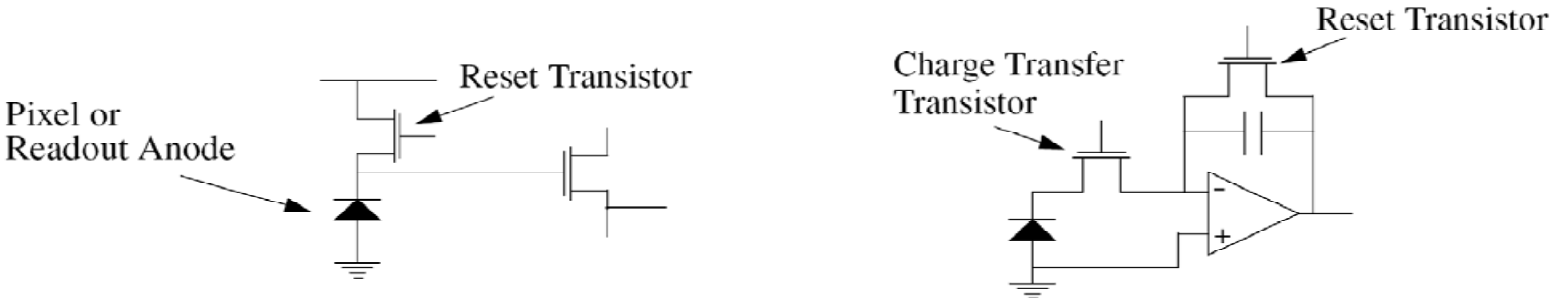
For $\left.\begin{array}{l}\tau = RC = 1\text{s},\\ t \qquad\quad = 1\mu\text{s}\end{array}\right\}$ $\sigma_v/(k_BT/C)^{1/2} \approx$ 4.5%

Pixel Reset, or Charge Transfer and kTC Noise vs Time

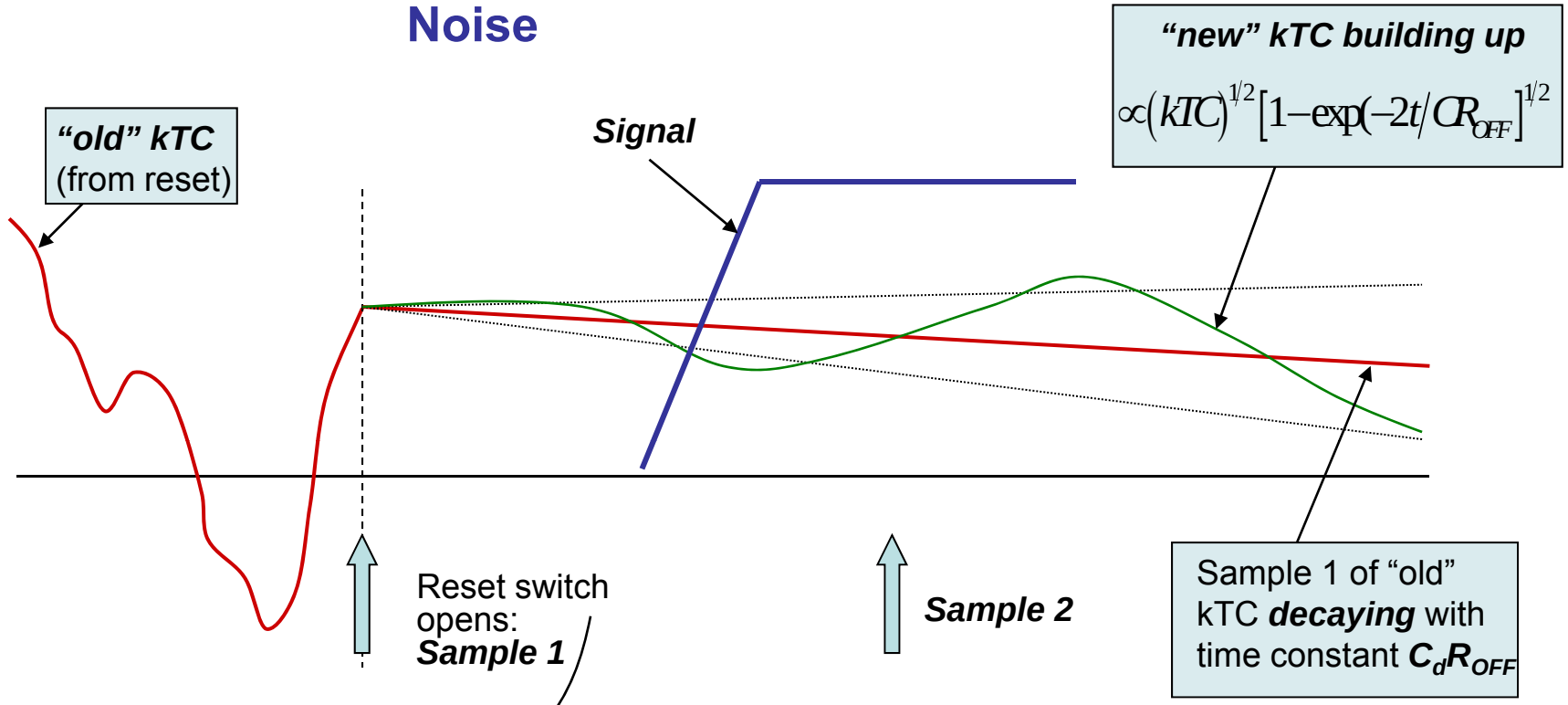
$$\sigma_q/(kTC)^{1/2} = (1 - e^{-2t/RC})^{1/2}$$

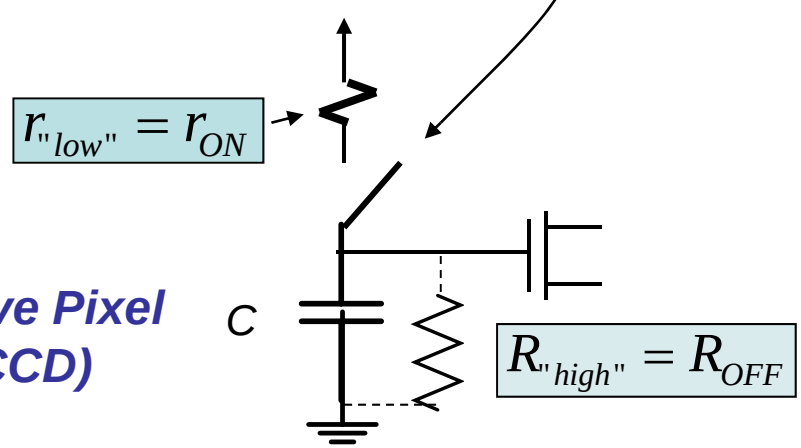






Correlated Double Sampling (CDS) and *kTC* Noise





Active Pixel (or CCD)

$$R_{OFF} / r_{ON} \sim 10^6 - 10^{11}$$

Example:

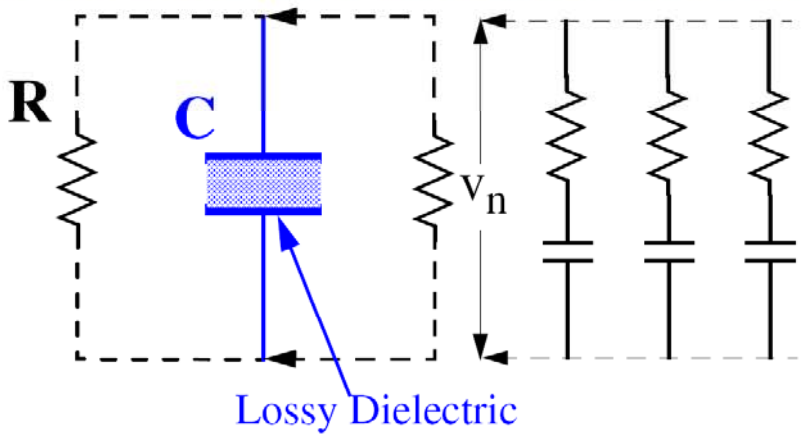
$C \sim 20$ fF

$r_{ON} \sim 10^3$ ohms $\quad r_{ON}C \sim 20$ ps

$R_{OFF} \sim 10^{13}$ ohms $\quad R_{OFF}C = 0.2$ s

8. Noise from Dielectrics

Lossy Dielectrics and Complex Networks



$$\varepsilon = \varepsilon' + j\varepsilon'' = \varepsilon'\left(1 + j\frac{\varepsilon''}{\varepsilon'}\right)$$

$$\frac{\varepsilon''}{\varepsilon'} = D = \frac{1}{Q} = \frac{g(\omega)}{\omega C}$$

From the Fluctuation-Dissipation Theorem: $\overline{v_n^2} = 4k_B T D \frac{1}{\omega C}$; $\overline{q_n^2} = 4k_B T C \frac{D}{\omega}$

For charge measurement (ns to ms range) $ENC_D \approx k_B T C(2.4D)$

For C = 1 pF at T = 293 K

$\sigma_q = (k_B\, T\, C)^{1/2} = 400$ e rms

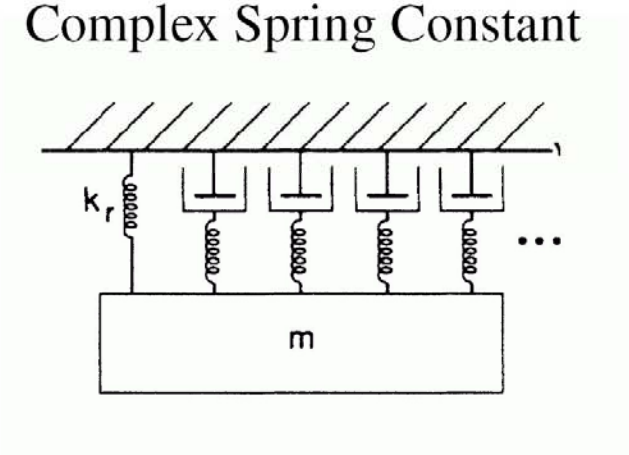
$ENC_a < 5$ e (with an optimized amplifier)

Dielectric Noise Contribution:

$D \approx 5\times10^{-5}$, $ENC_D \approx 4$ e rms

$D \approx 2\times10^{-2}$, $ENC_D \approx 86$ e rms (G10 circuit board)

Complex Spring Constant



$k = k_r(1 + j\varphi)$

k_r = "relaxed" spring constant

$f = 1/Q \approx 10^{-4} - 10^{-8}$

$$v_n^2(\omega) = 4k_B T \frac{R(1 + CDR\omega)}{C^2R^2\omega^2 + (1 + CDR\omega)}$$

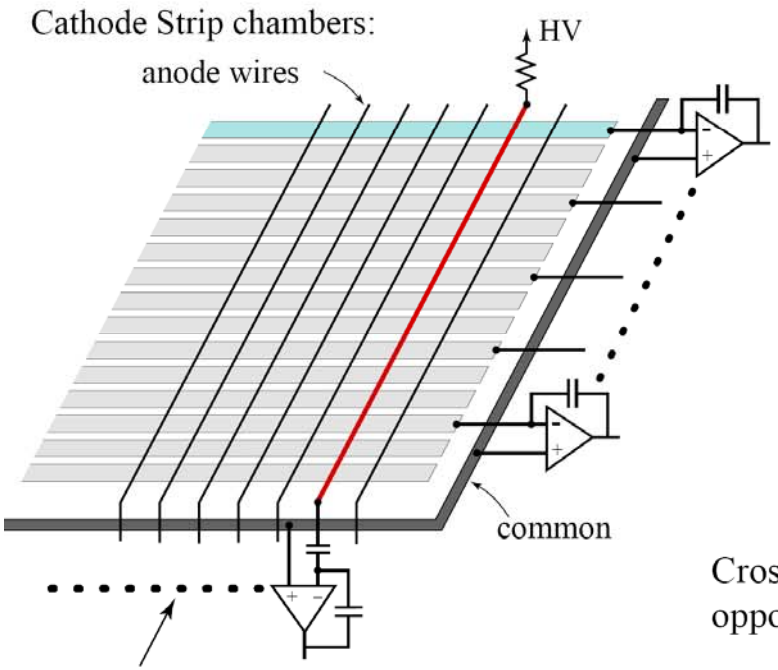
$$\sigma_v^2 = \int_0^{f_h} v_n^2(f)\, df \cong \frac{k_B T\left(\pi + \frac{D^2}{\pi R C f_h(1+D^2)} + Log(4\pi^2R^2C^2[1+D^2])\right)}{\pi C(1+D^2)} + \frac{2k_B T Log(f_h)}{\pi C(1+D^2)}$$

$$\sigma_v^2 \approx \frac{k_B T}{C}$$

9. Signal Return in Multi-Element Detectors

Signal Return in Multi-Element Detectors

Cathode Strip chambers:

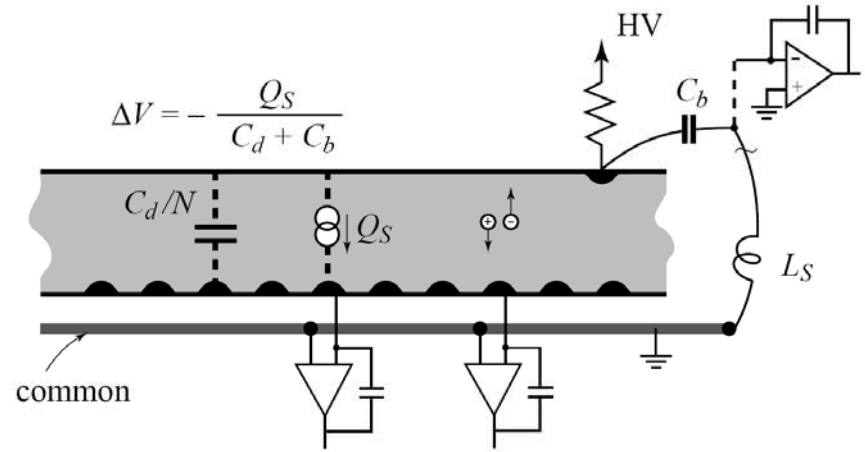


Pulse shaping to attenuate reflection:

$$t_p \gtrsim 4\ t_D$$

t_D – propagation delay in the longer electrode

$t_p \gtrsim 25$ns for $l = 2$m

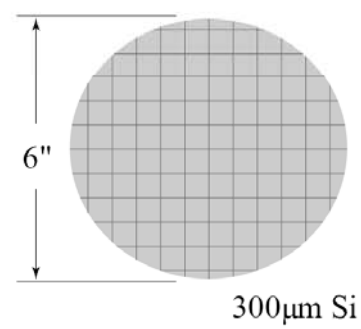


Crosstalk into channel due to opposite electrode feedback:

$$Q_i = -Q_S \frac{C_d / N}{C_d + C_b}$$

N = number of strips or pixels

- for $C_b \to 0$, $Q_i = -Q_S / N$
- for single particles and large N bypass not needed (except for shielding)
- for showers or high occupancy, C_b is essential.



$C_d \approx 5.4$ nF

$C_b > 10\ C_d$!

L_S important !

Back side readout → high noise!

10. Literature

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Appendix

A. 1
Noise Calculation: Time Domain and Frequency Domain

- For <u>time invariant</u> systems impulse response <u>h(t)</u> and weighting function <u>w(t)</u> are interchangeable for noise calculations.

From Parseval's theorem, noise variance can be expressed as:

$$\sigma^2 = \bar{n}\, q^2 \int_{-\infty}^{\infty} h^2(t)dt = 2\,\bar{n}\, q^2 \int_{0}^{\infty} |H(\omega)|^2\, df$$

Campbell's theorem

Also: $\sigma^2 = W_o \int_{0}^{\infty} |H(\omega)|^2\, df$

Thus noise spectral density: $W_o = 2\,\bar{n}\, q^2 = 2q_e I_o$ for shot noise
(similarly $W_o = 4kT/R$ for thermal noise)

<u>Parallel noise integral</u>:

I_2 $$\overline{ENC_p^{\,2}} = \tfrac{1}{2} W_{op} \int_{-\infty}^{\infty} h^2(t)dt = W_{op} \int_{0}^{\infty} |H(\omega)|^2 df$$

parallel noise (current) spectral density = W_{op}

<u>Series noise integral</u>:

I_1 $$\overline{ENC_s^{\,2}} = \tfrac{1}{2} W_{os}\, C_{in}^{\,2} \int_{-\infty}^{\infty} h'^2(t)dt = W_{os} C_{in}^{\,2} \int_{0}^{\infty} \omega^2 |H(\omega)|^2\, df$$

series noise (voltage) spectral density, $W_{os} = \overline{e_n^{\,2}} = 4kTR_s$

(since $\int_{-\infty}^{\infty} h'^2(t)dt = 2\int_{0}^{\infty} \omega^2 |H(\omega)|^2\, df$)

- For <u>time variant</u> systems, noise is calculated using <u>weighting function</u> and time domain relations.

A.2 Parallel Noise due to ***Dark Current*** in Devices with ***Avalanche Gain***

Parallel noise (from Campbell's theorem):

$$ENC_p^2 = nq^2 \int_{-\infty}^{\infty} w^2(t)dt = nq^2 t_G$$

$$nq^2 = q(nq) = qI_o$$

n = rate of charge impulses

t_G = equivalent integration time

- Devices ***without avalanche gain***: $\boldsymbol{q = e}$ = electron charge
- Devices ***with avalanche gain*** M : $\boldsymbol{q = Me}$ = charge in each impulse in Poisson sequence

Parallel noise due to dark current in avalanche photo-diodes:

$$ENC_p^2 = n(Me)^2 t_G$$

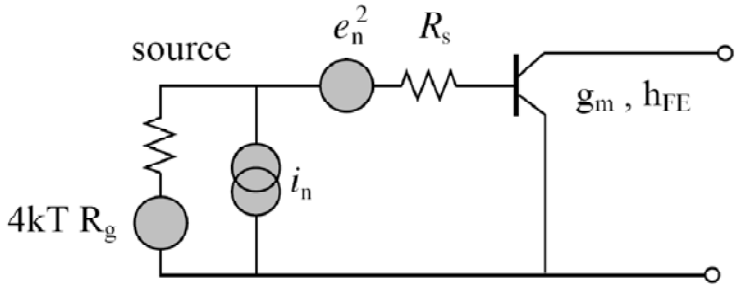
$$ENC_p^2 = I_o M \cdot e\, t_G$$

Avalanche gain magnifies the noise (ENC_p) by $\boldsymbol{M^{1/2}}$ compared to the shot noise from a current I_o ,

i.e., the noise due to ***avalanche dark current*** $\boldsymbol{I_o}$ equals the noise due to single electron current $\boldsymbol{MI_o}$.

(Surface leakage current noise is not subject to avalanche gain.)

A3. Noise Figure vs Noise Voltage, Noise Current and Noise Temperature



$R_s = 1/2g_m + r_{bb'}$
$i_n^2 = 2\ eI_b = 2e\ \ I_c/h_{FE}$

$NF = 10 \log_{10} [(\text{source} + \text{ampl.})/\text{source}]$

$NF = 10 \log_{10} [1+(e_n^2 + i_n^2 R_g^2)/4kTR_g]$

For $i_n^2 R_g^2 << e_n^2$, i.e., $R_g = 50$:

$\underline{NF = 10 \log_{10} [1 + R_s/R_g]}$ $= 10 \log_{10} [1 + T_n/T]$ [db]

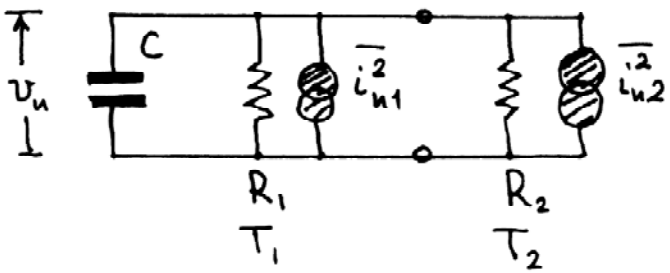
T_n = ampl. noise temperature

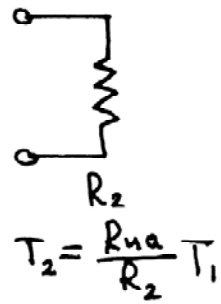
$R_s T = R_g T_n \rightarrow T_n/T = R_s/R_g$

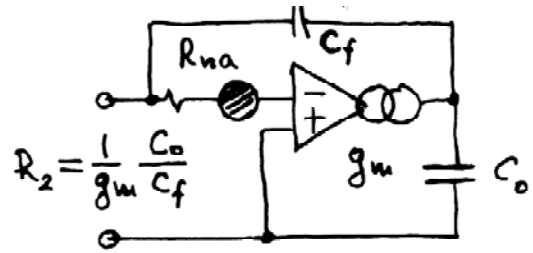
NF [db]	R_s [ ]	e_n[nV/Hz$^{1/2}$]	T_n[K]
3	50	0.91	293
2	29	0.71	170
1	13	0.47	76
0.5	6	0.32	35

<u>ENC</u> from <u>e_n</u> , <u>i_n</u> (with $C_D + C_A$ and t_m)

A4. Noise in Resistors at Different Temperatures and "Electronic Cooling" by Feedback







Total fluctuation (all frequencies):

$$\sigma_v^2 = \frac{k_B}{C} \cdot R_1 \| R_2 \left(\frac{T_1}{R_1} + \frac{T_2}{R_2} \right)$$

1. for $T_1 = T_2 \rightarrow \sigma_v^2 = \frac{k_B T_1}{C}$

2. if $T_2 = 0 \rightarrow \sigma_v^2 = \frac{k_B T_1}{C} \cdot \frac{1}{1 + R_1 / R_2}$

With feedback (cooled) damping:

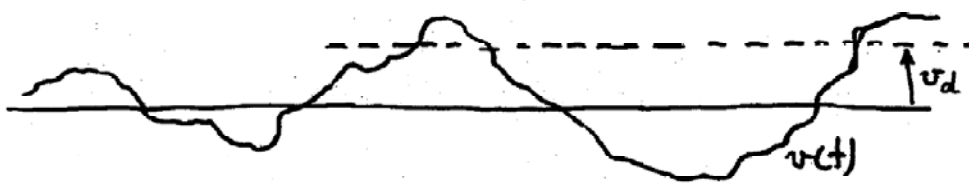
$$\sigma_v^2 = \frac{k_B T_1}{C} \cdot \frac{1}{1 + R_1 / R_2} \left(1 + \frac{R_{na} R_1}{R_2^2} \right)$$

σ_v min for $R_2 = (R_1 R_{na})^{1/2}$

$$\rightarrow \sigma_v^2 = \frac{kT_1}{C} \cdot \frac{2}{1 + (R_1 / R_{na})^{1/2}}$$

$$\left.\begin{array}{l} R_1 = 10^6 \Omega \\ R_{na} = 100 \Omega \end{array}\right\} \begin{array}{l} R_2 = 10^4 \Omega \\ \sigma_v^2 \cong \frac{k_B T_1}{C} \cdot \frac{1}{50} \end{array}$$

A5. Zero-Crossing Statistics of Noise



Discriminator threshold

$W(\omega) =$ spectral density

$K(\tau) =$ autocorrelation function

$\sigma = [K(0)]^{1/2} =$ rms noise

Rate of ***positive*** zero crossings:

$$n_{zc+} = \frac{1}{2\pi}\left[-\frac{K''(0)}{K(0)}\right]^{\frac{1}{2}} = \frac{1}{2\pi}\left[\frac{\int_0^\infty \omega^2 W(\omega)d\omega}{\int_0^\infty W(\omega)d\omega}\right]^{\frac{1}{2}}$$

Rate of positive level crossings:

$$n(v_d)_+ = n_{zc+} \cdot \exp\left[-v_d^2 \big/ 2K(0)\right]$$

Example: 2nd order high frequency cutoff (2 RC integrations)

Solution:

$$n_{zc+} = \frac{1}{2\pi(\tau_1\tau_2)^{1/2}}$$

For: $\tau_1 = \tau_2 = 50ns; 1\mu s$

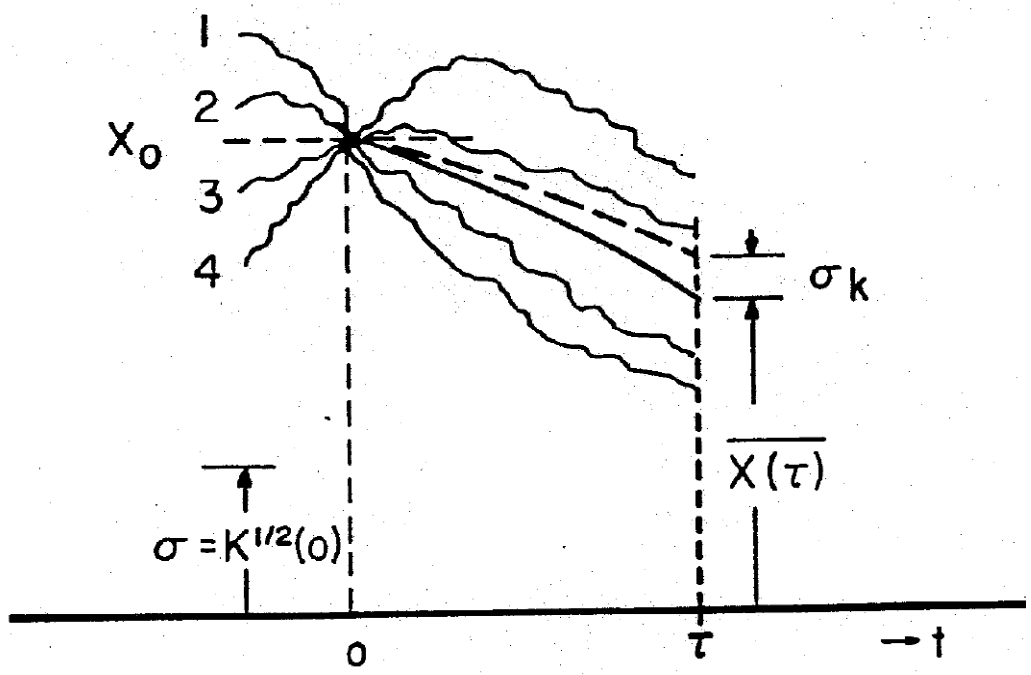
$n_{zc+} = 3MHz; 150kHz$

Threshold crossing rate:

$v_d/\sigma \Rightarrow$	$n(v_d)_+/n_{zc+}$
2	1.4×10^{-1}
3	1.1×10^{-2}
4	3.3×10^{-4}
5	4×10^{-6}

J.S. Bendat, Principles and Applications of Random Noise Theory, J. Wiley&Sons, 1958,p. 125

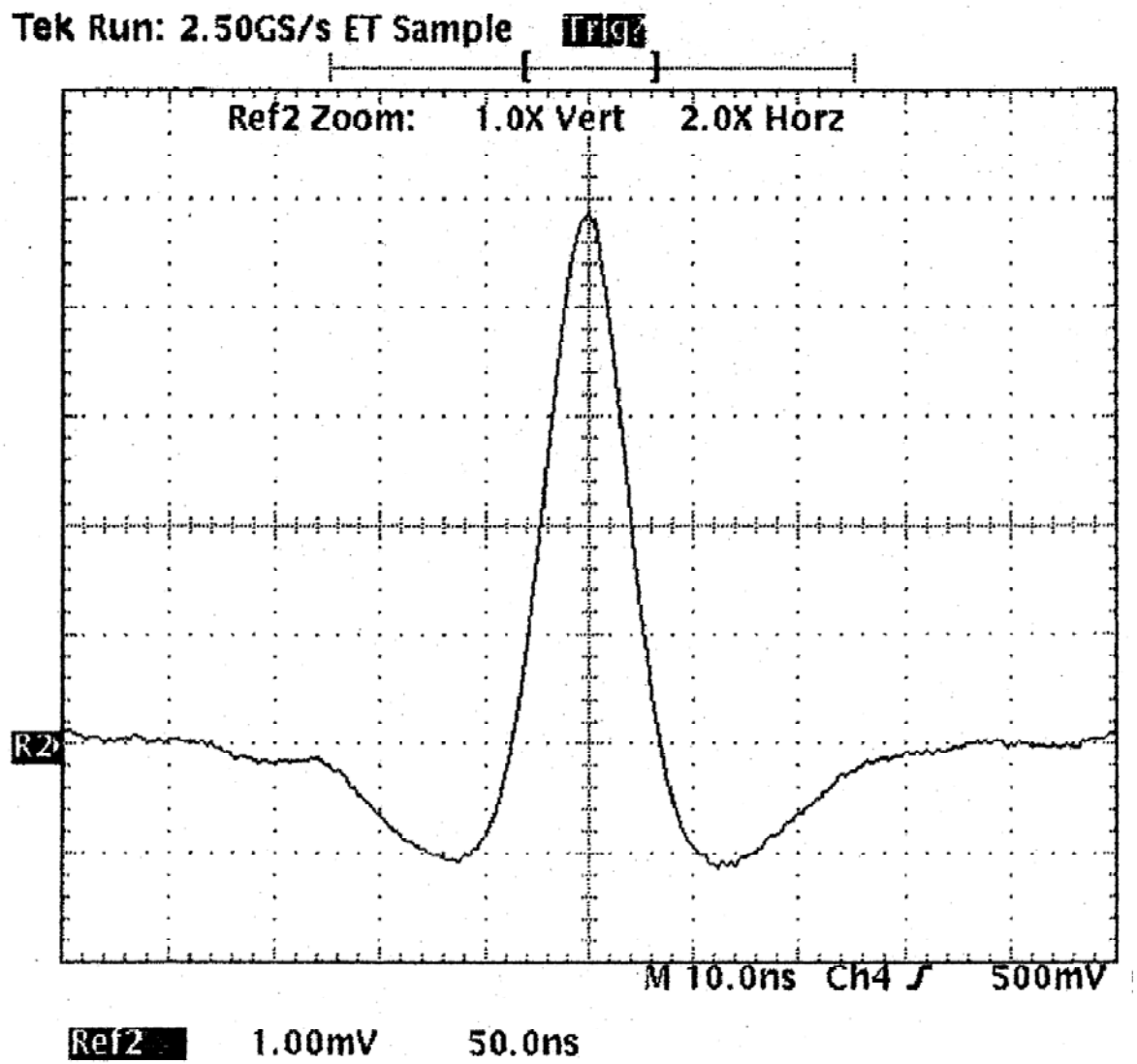
A.6 Autocorrelation Function as a Diagnostic Tool



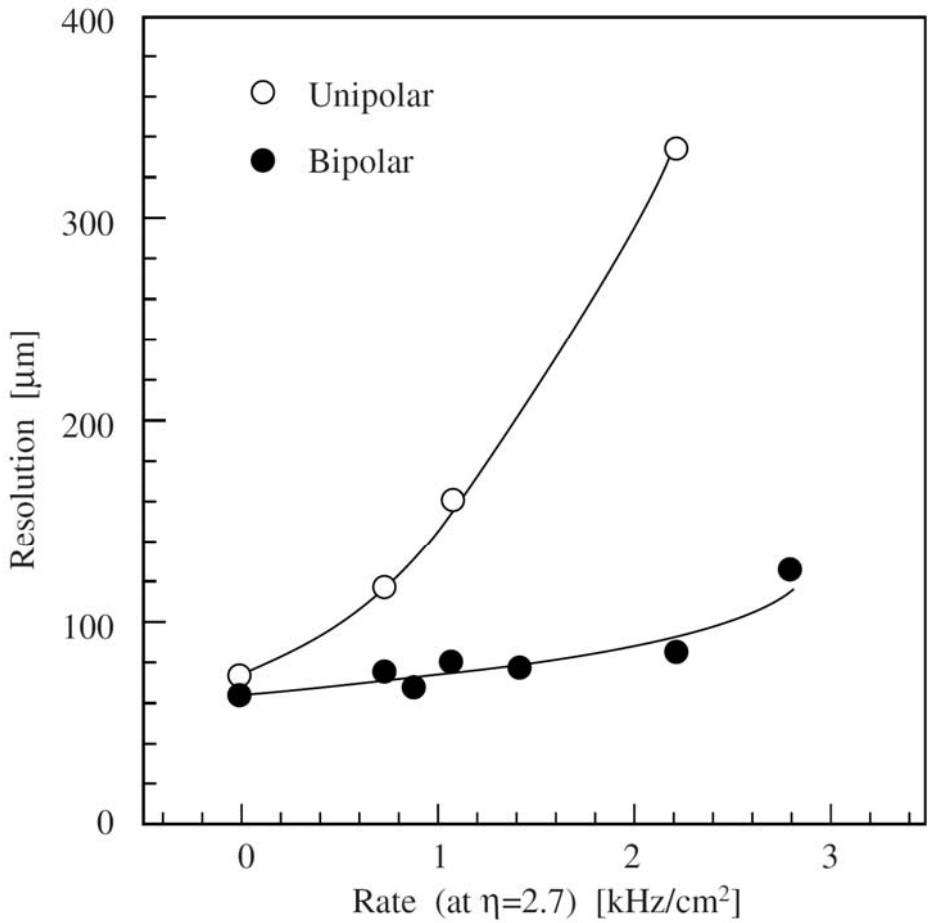
$$\frac{\overline{X(\tau)}}{X_0} = \frac{K(\tau)}{K(0)} = k(\tau)$$

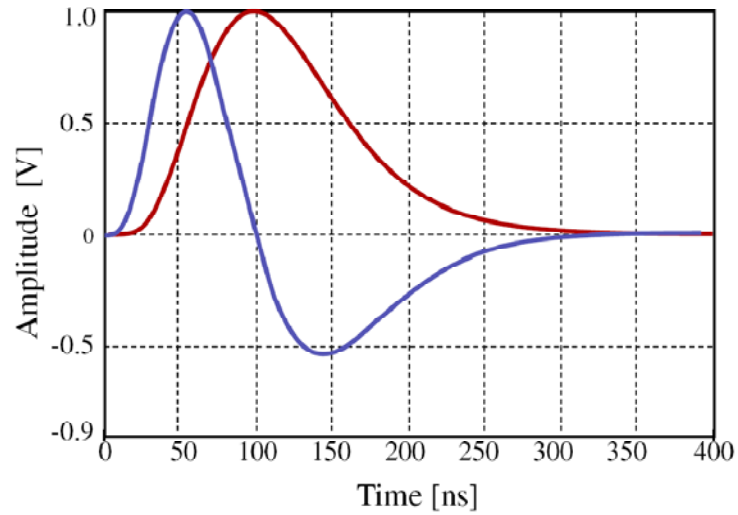
$$\sigma_k^2 = K(0) - \frac{K^2(\tau)}{K(0)} = \sigma^2\left[1 - k^2(\tau)\right]$$

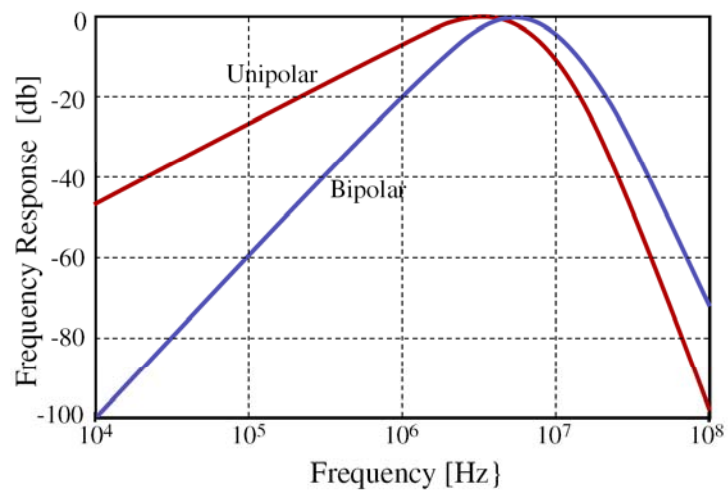
An Example of Noise Autocorrelation Measurement by Digital Oscilloscope



A.7 Pileup Effect on Centroid Position Resolution for Unipolar and Bipolar Shaping (Weighting Function)







Detector: 1.5 m long MWPC with interpolating cathode strip readout; Peaking time ~ 250 ns